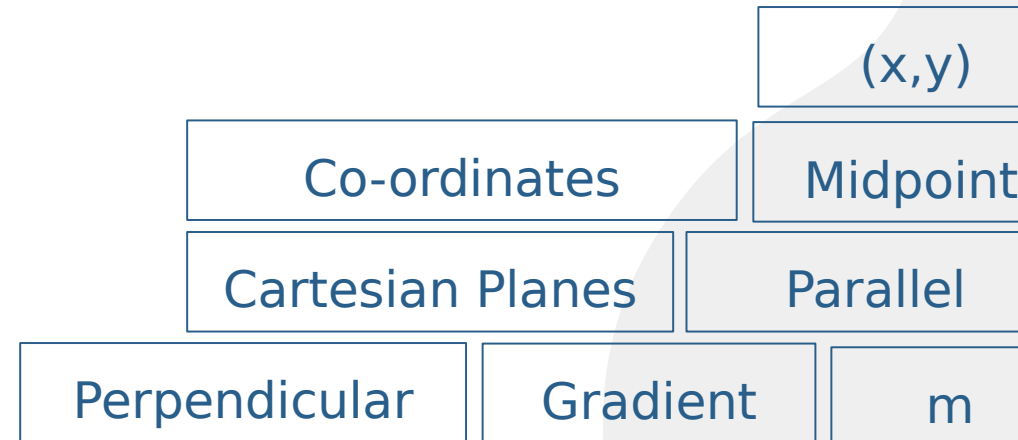


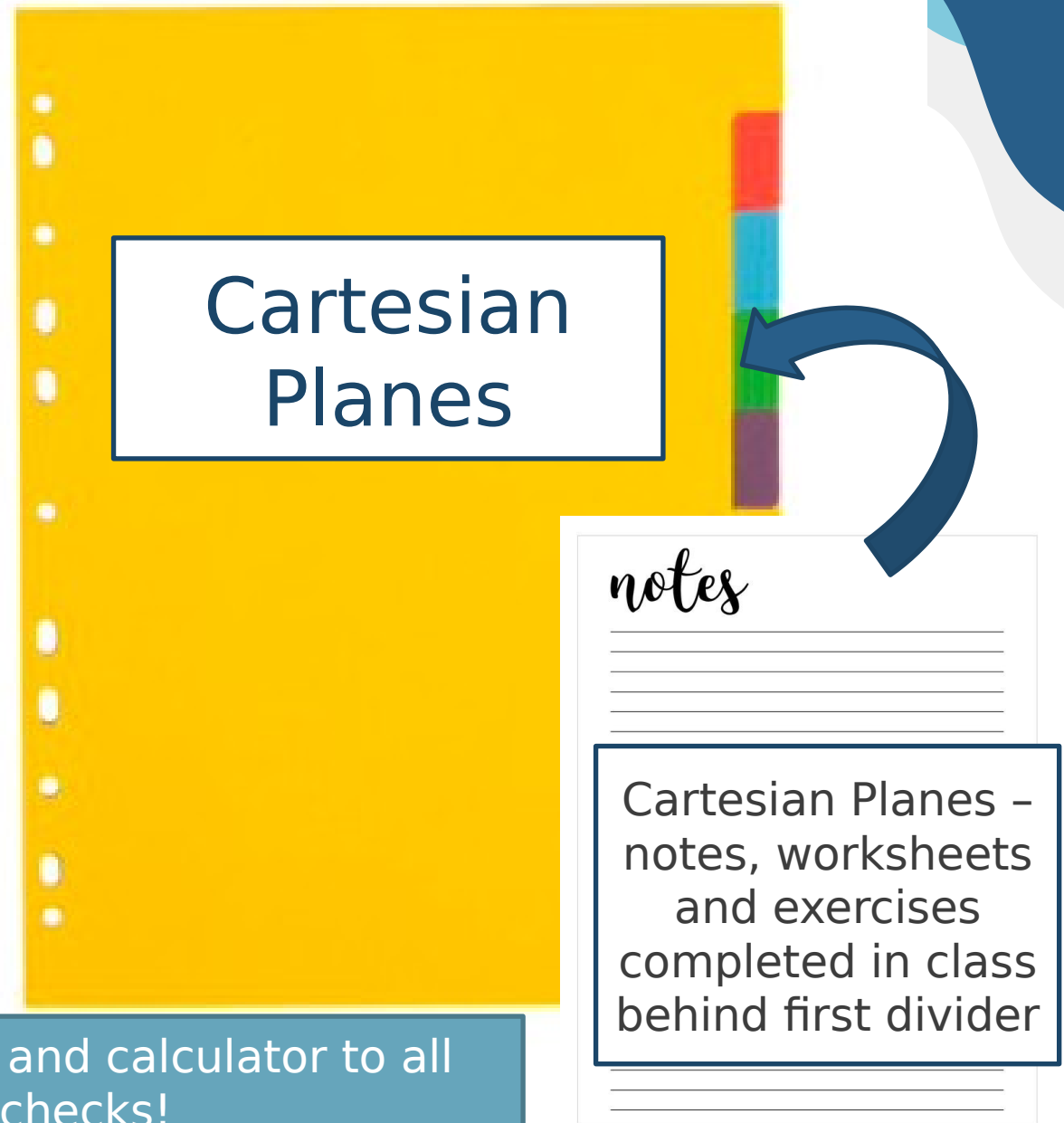
Cartesian Planes

IGCSE Y10 Mathematics

0607



Math file
Fourth divider
should be named
Cartesian Planes
and all notes and
worksheets from
this section
placed behind
this divider



Bring your file, extra paper, stationery and calculator to all classes – there will be file checks!

Core:

- Plotting of points and reading from a graph in the Cartesian plane
- Solution of simultaneous linear equations in two variables
- Distance between two points
- Mid-point of a line segment
- Gradient of a line segment
- Gradient of parallel lines
- Equation of a straight line as $y = mx + c$ or $x = k$

Extended:

- Gradient of parallel and perpendicular lines
- Equation of a straight line as $y = mx + c$ and $ax + by = d$

Starter: Plot this co-ordinates as best you can!

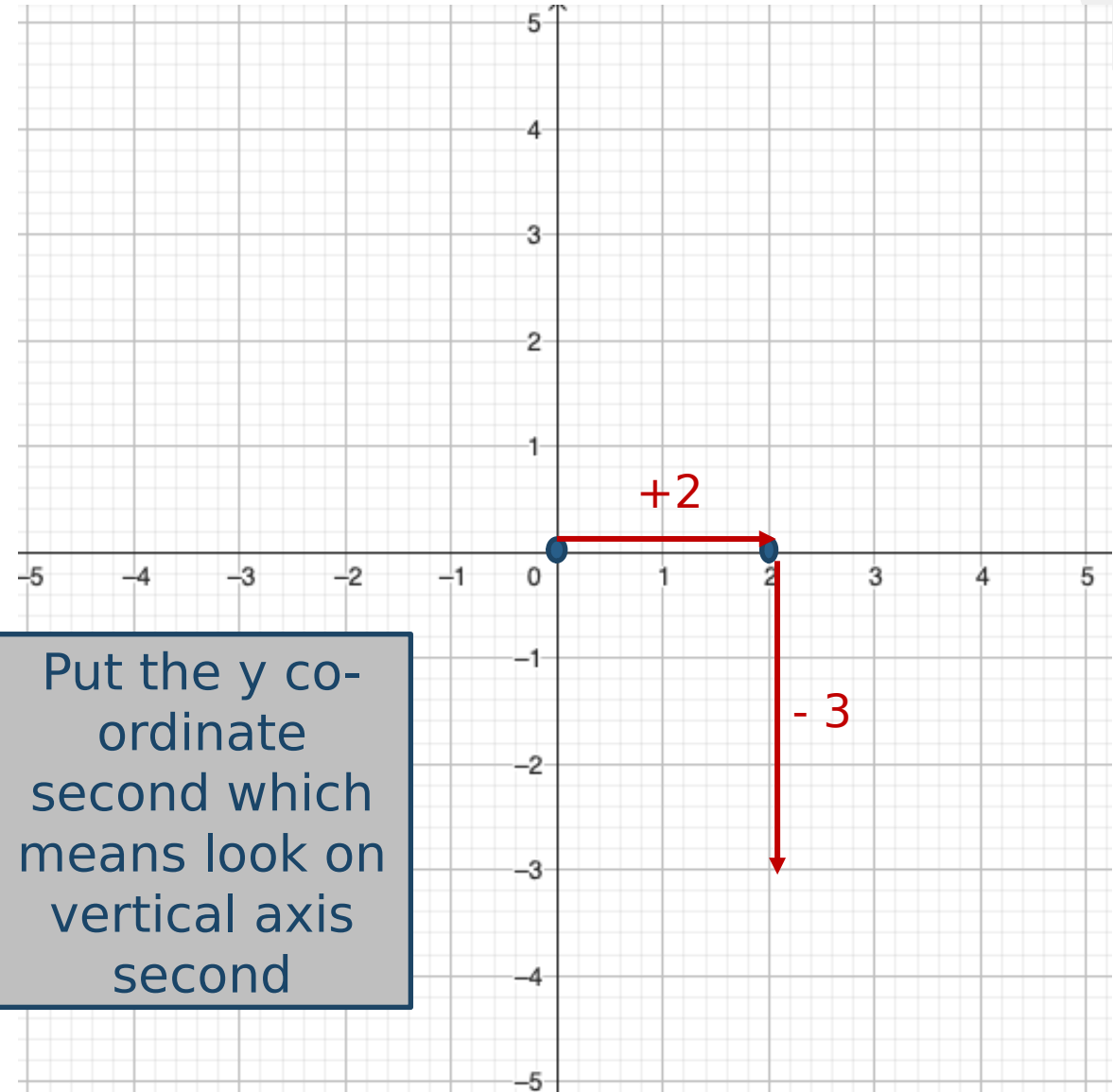
<input type="radio"/>	A = (1, -3)
<input type="radio"/>	B = (1, 3)
<input type="radio"/>	C = Point(xAxis) → (7, 0)
<input type="radio"/>	D = (5, 4)
<input type="radio"/>	E = (-3, 2)
<input type="radio"/>	F = (3, 4)
<input type="radio"/>	G = (3, -4)
<input type="radio"/>	H = (-3, -4)
<input type="radio"/>	I = (-3, 4)

Co-ordinates

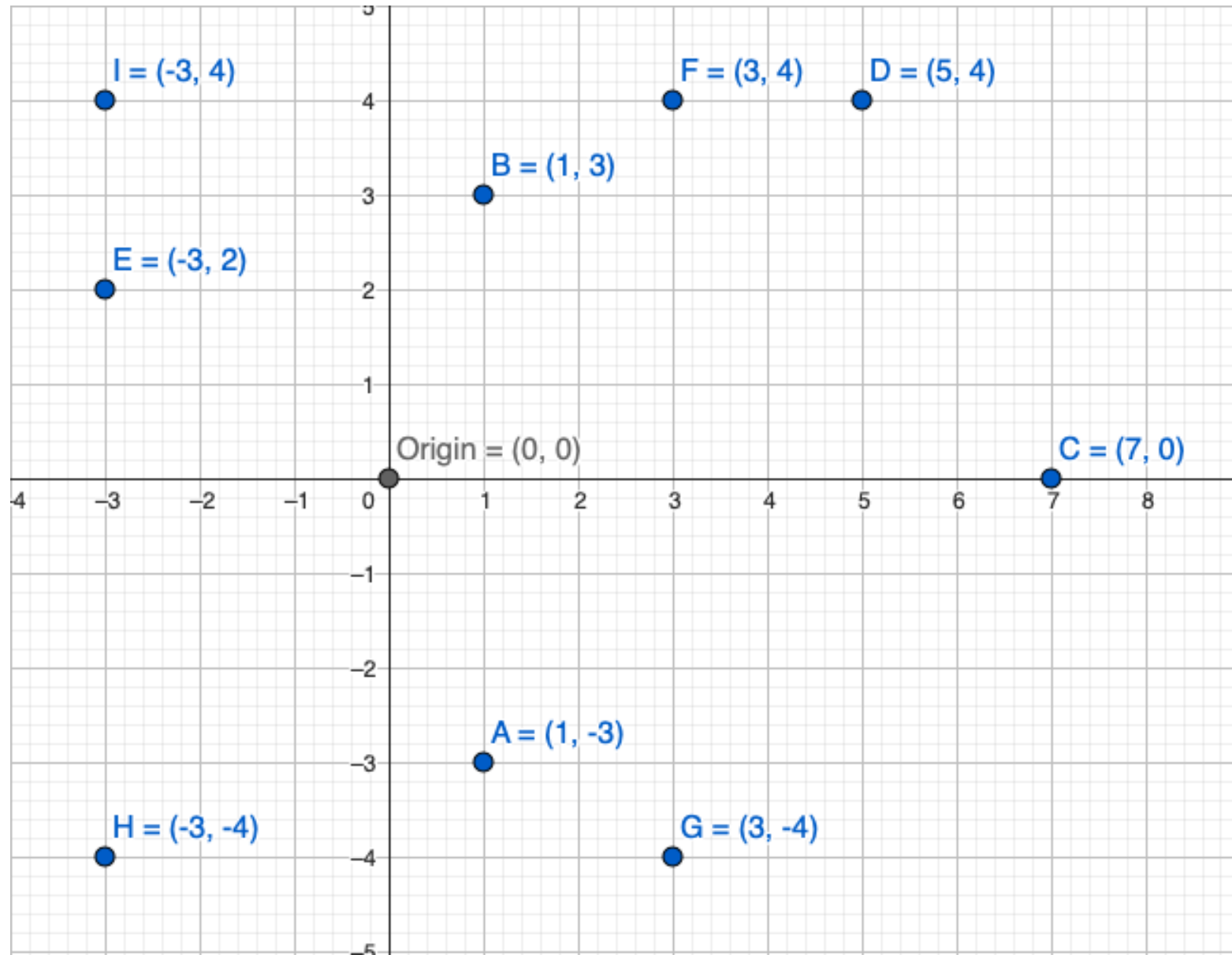
- When we want to plot a point on a graph we need to know where to put it
- If the horizontal axis is labelled **x** and the vertical axis is labelled **y**, then the **x** and **y coordinates** are how far we go along the **x** and **y** axes to plot the

Put the x co-ordinate first which means look on horizontal axis first

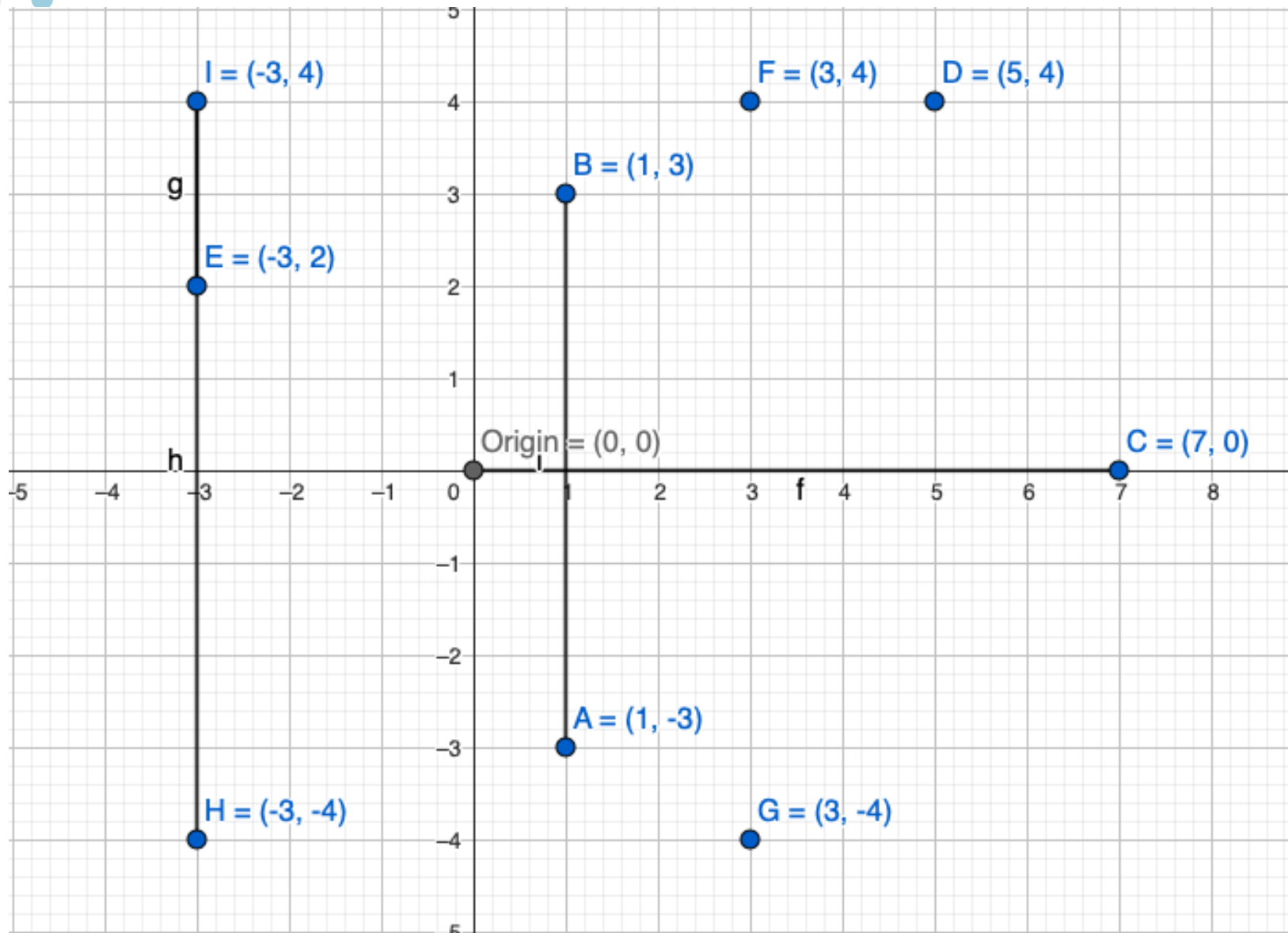
Put the y co-ordinate second which means look on vertical axis second



Check your starter



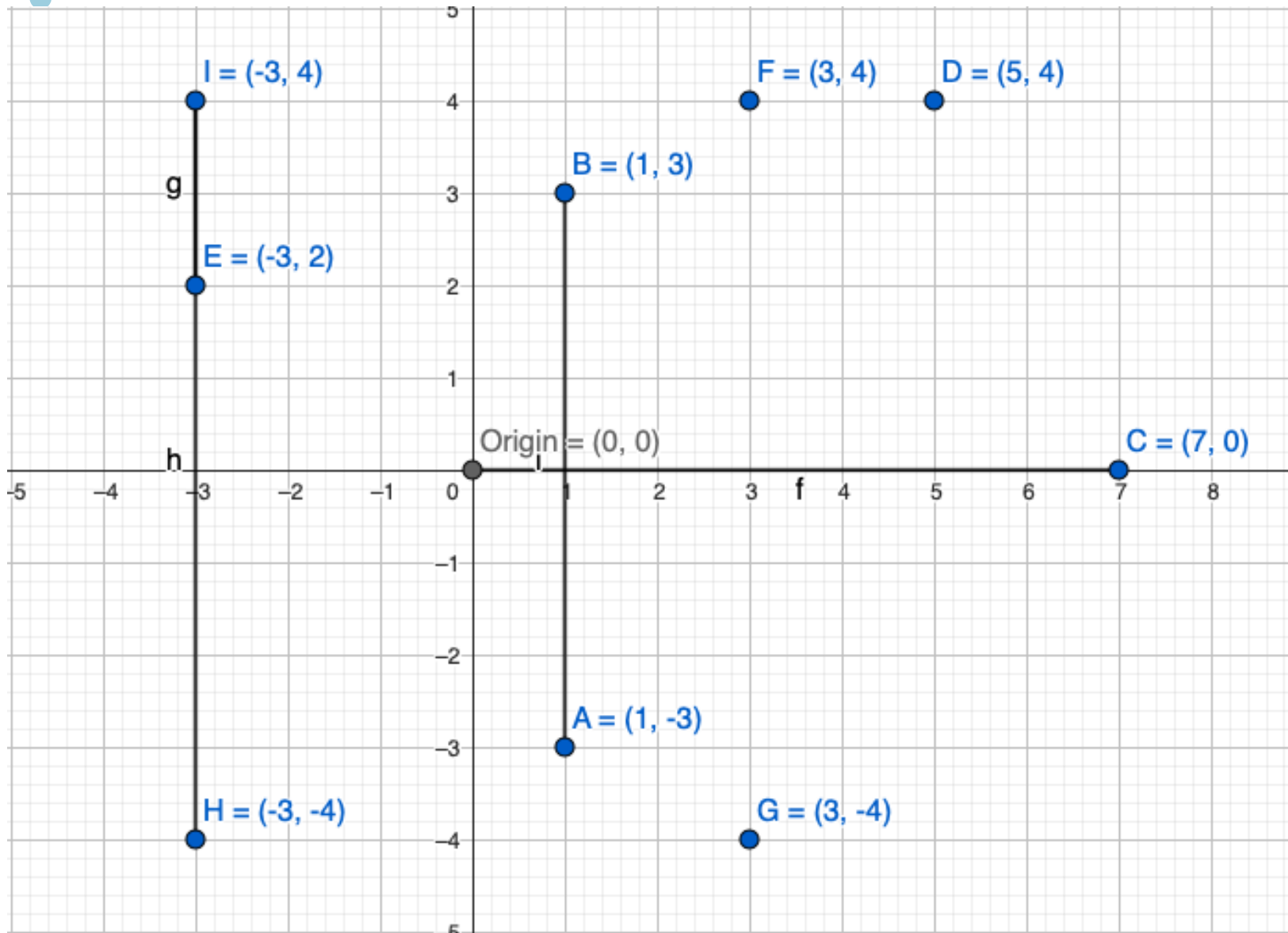
What is the distance between and



Origin
and
Point C?

7 units

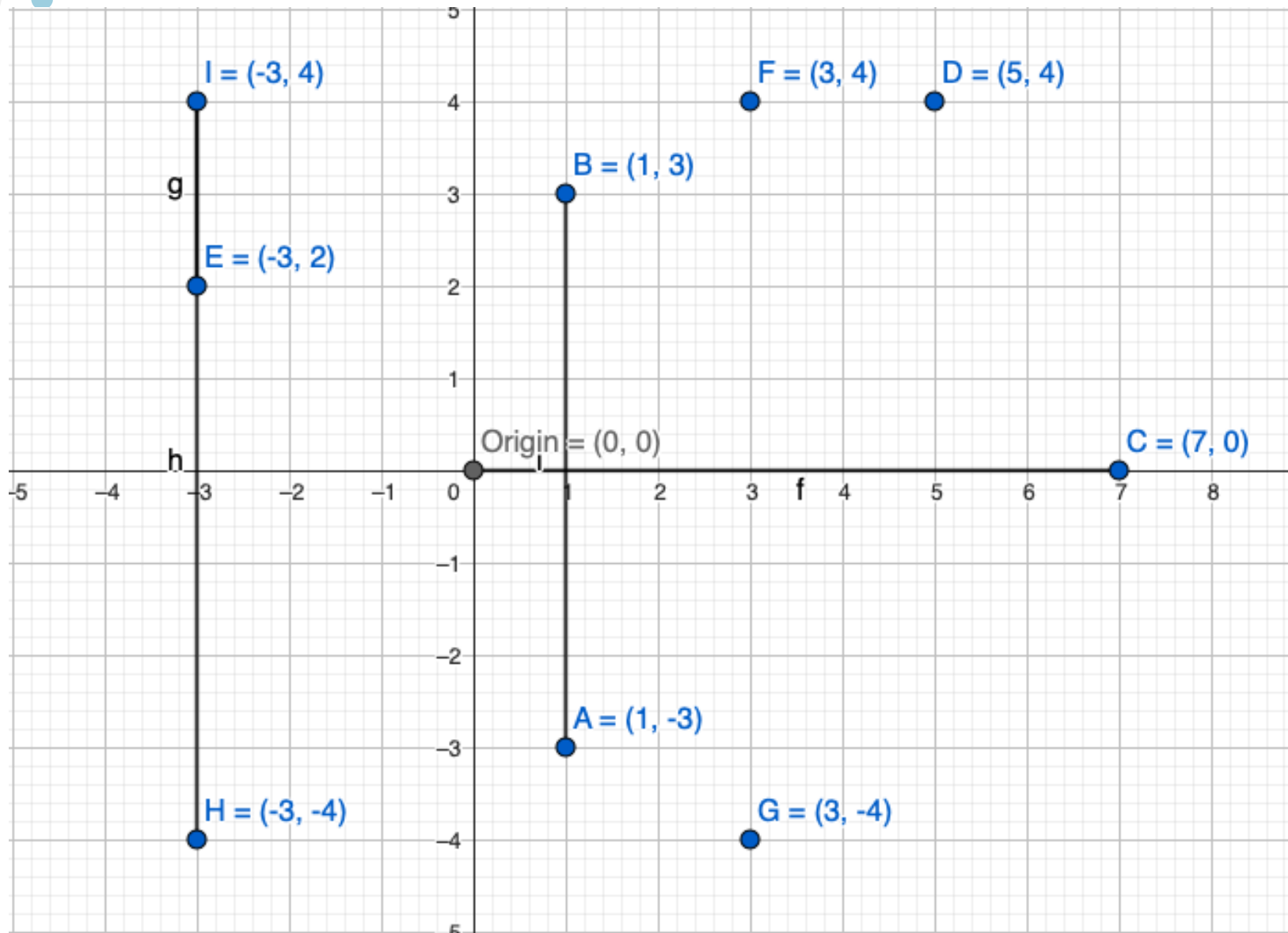
What is the distance between and



Point 1
and
Point E?

2 units

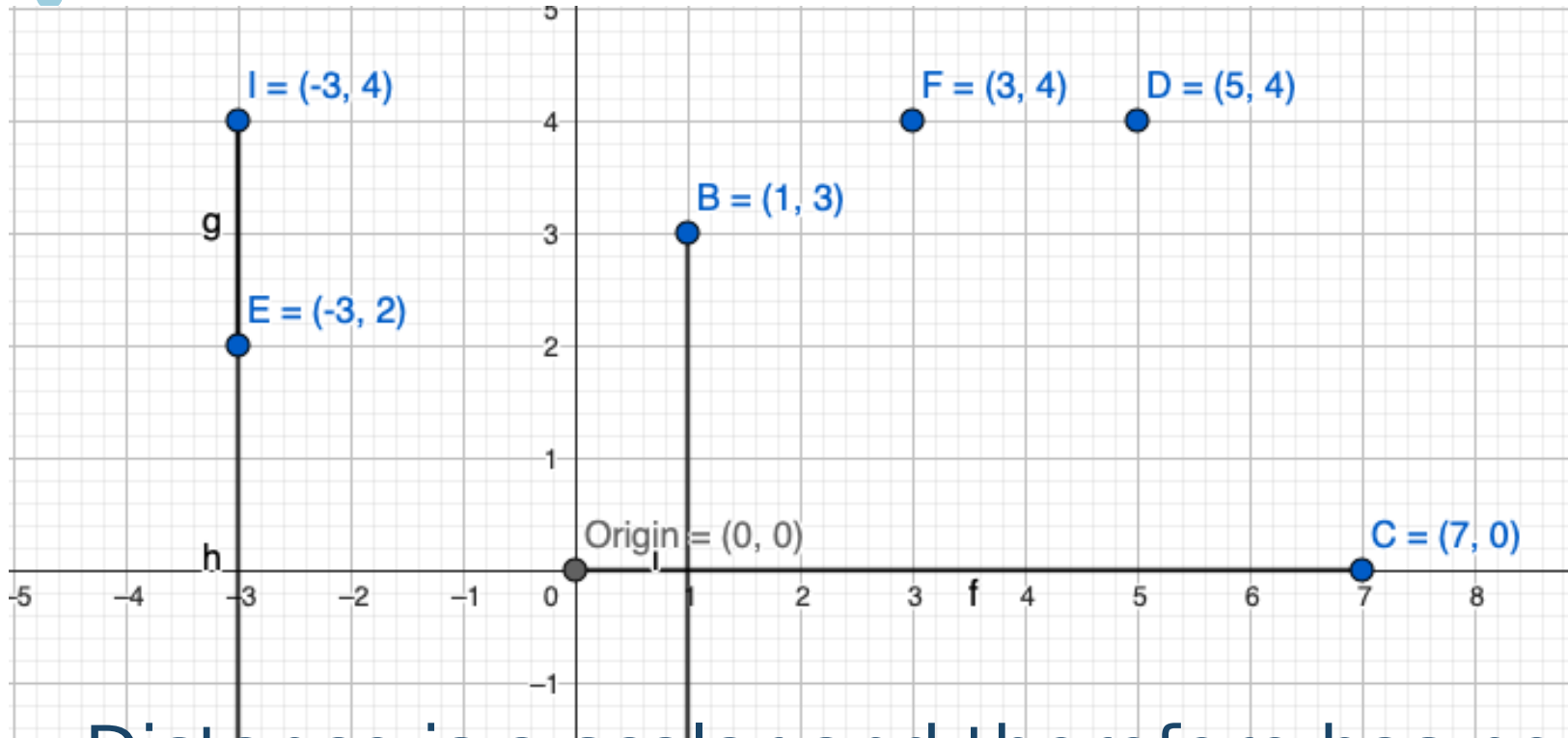
What is the distance between and



Point B
and
Point A?

6 units

What is the distance between and

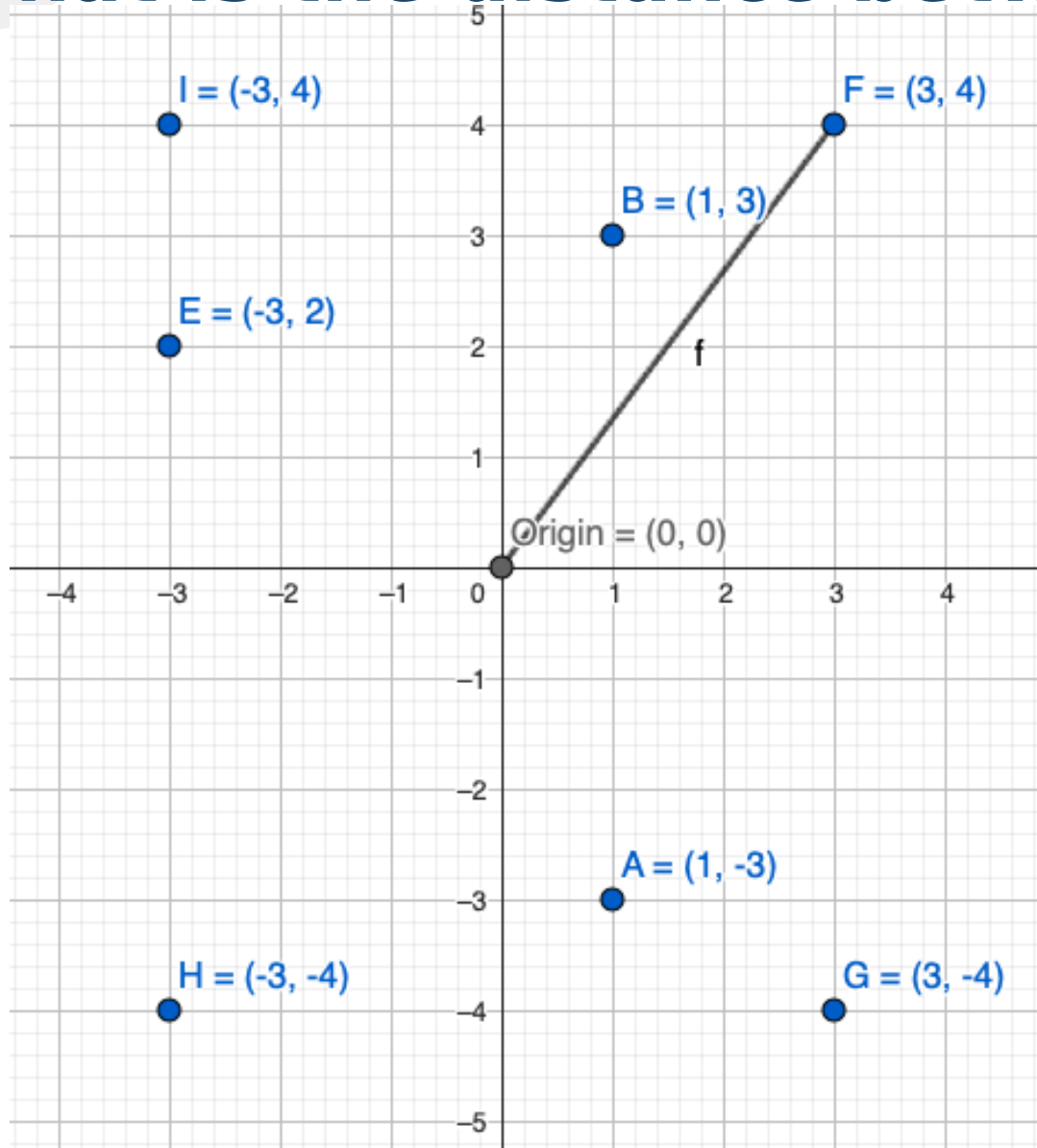


Point 1
and
Point H?

8 units

Distance is a scalar and therefore has no direction. Notice when you are giving the distance, you are only saying how long it is, not how long it is in a particular direction

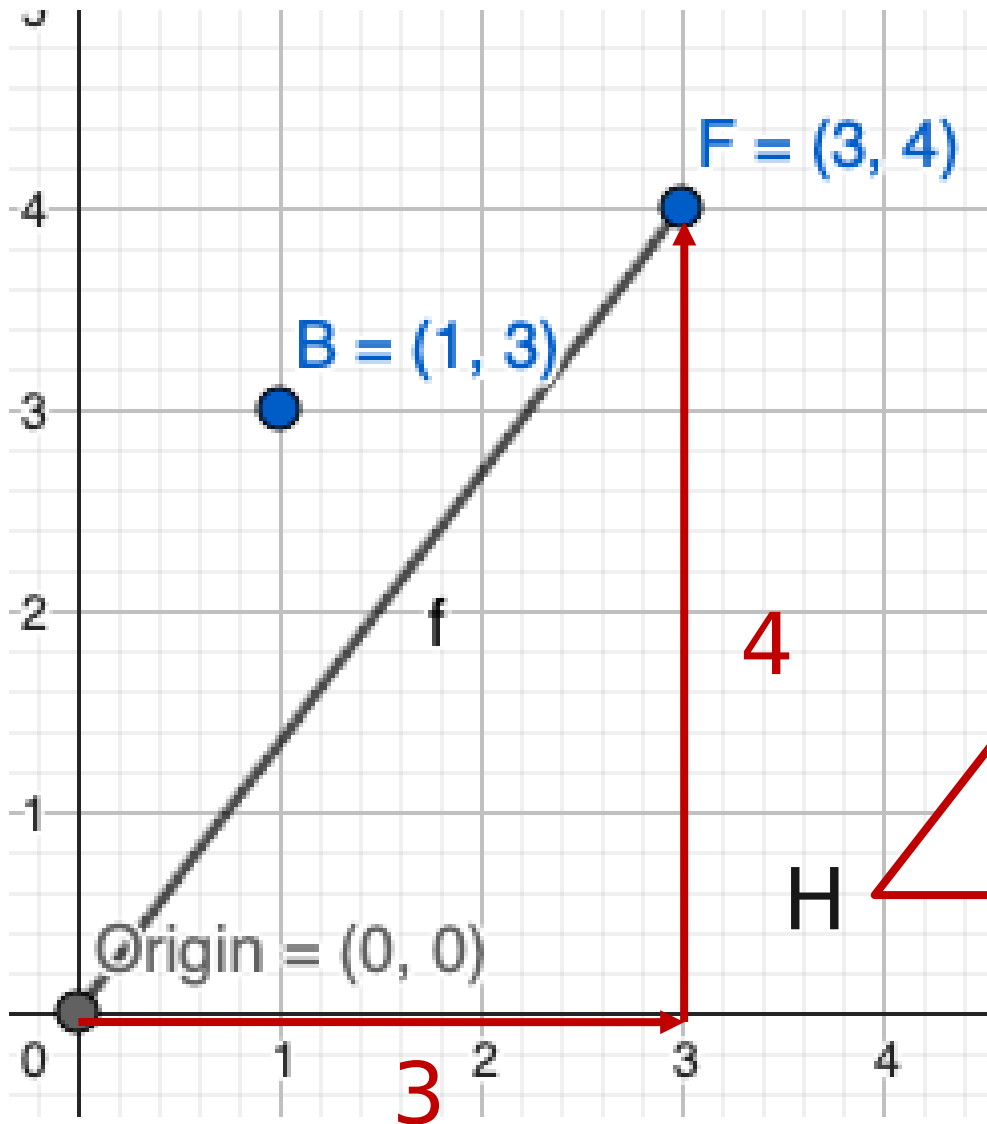
What is the distance between and



Point O
and
Point F?

How would
you be able
to figure this
out?

What is the distance between and



Use

Pythagoras!

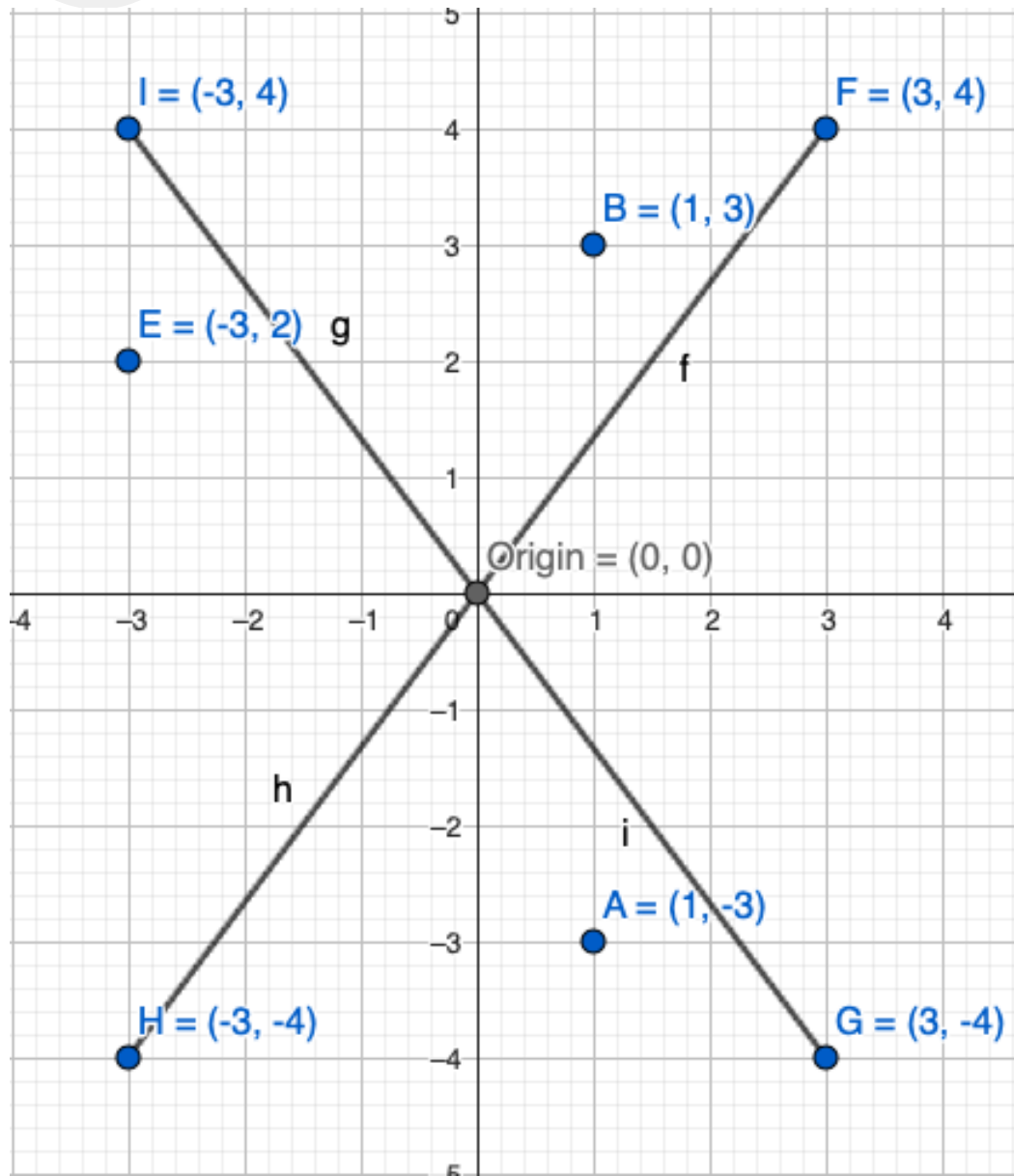
$$FH^2 = FG^2 + GH^2$$

$$FH^2 = 4^2 + 3^2$$

$$FH^2 = 25$$

$$FH = 5$$

What is the distance between and



Origin
and
Point F?

5
units

Point F
and
Point H?

10
units

What is the distance between and

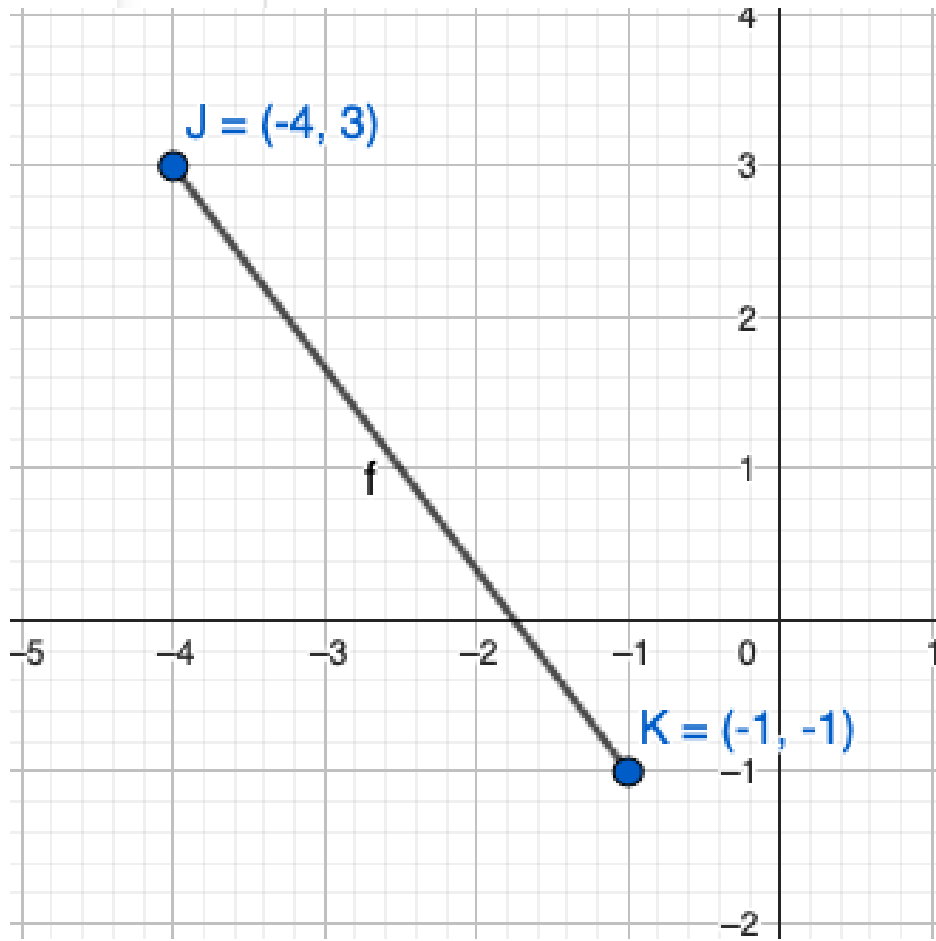


J = (-4, 3)



K = (-1, -1)

< Plot these co-ordinates

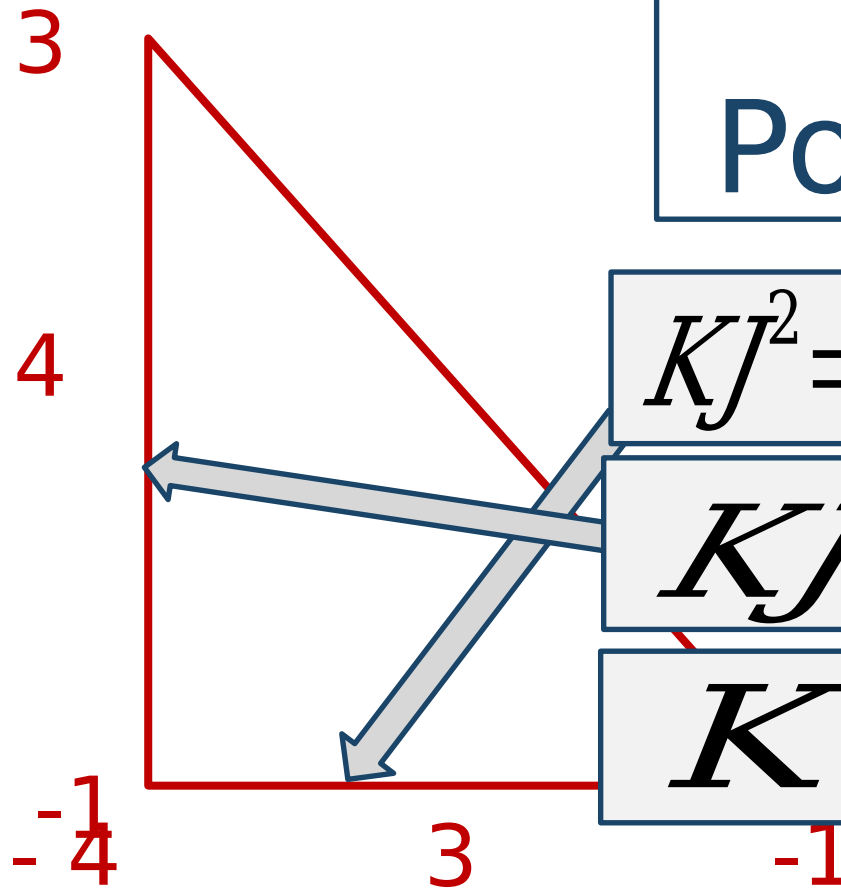


Point J
and
Point K?

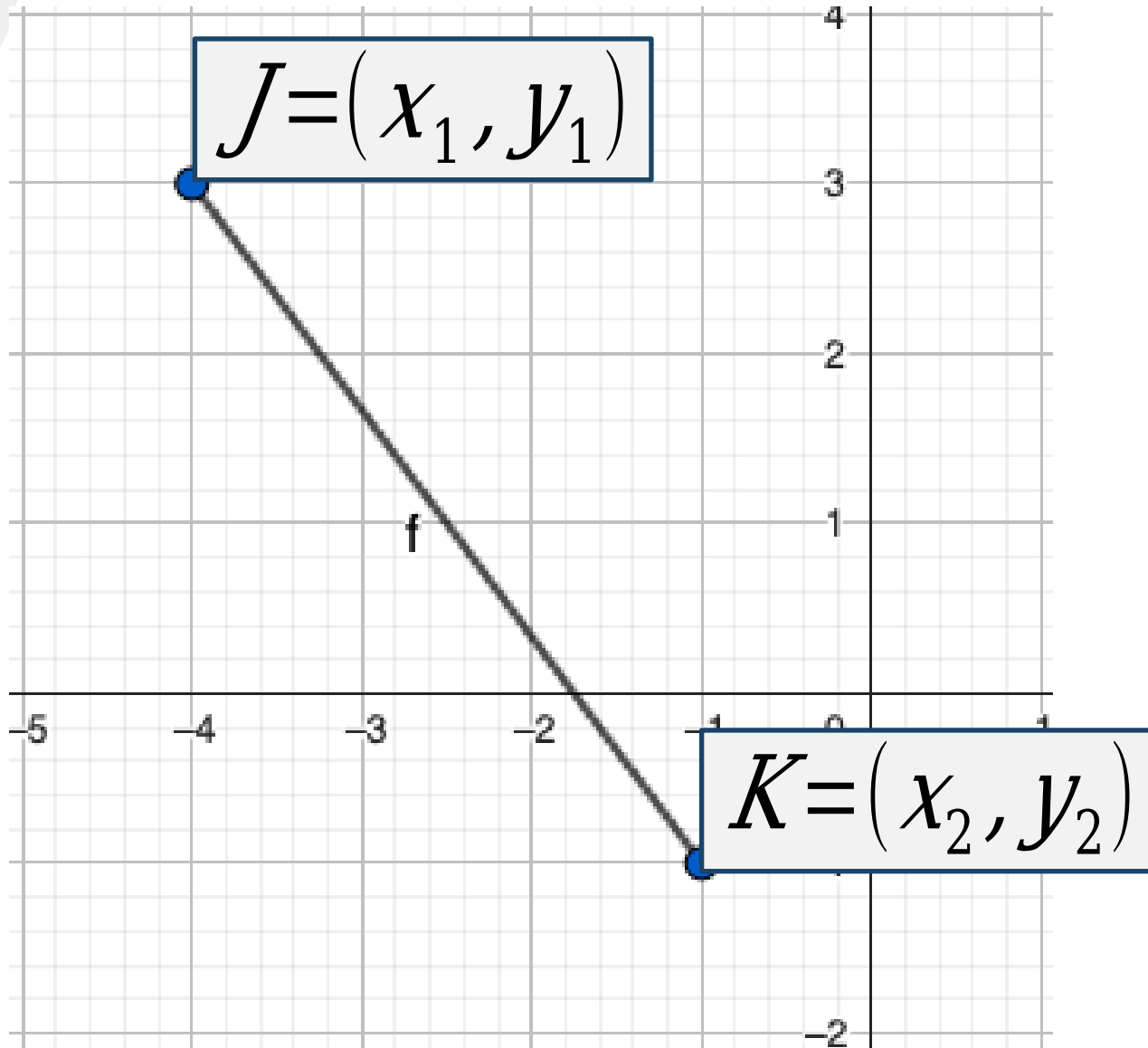
$$KJ^2 = (-4)^2 + 3^2$$

$$KJ^2 = 25$$

$$KJ = 5$$



What is the distance between and



Can you think of
a formula to find
distance KJ?

3 minutes

What is the distance between and

Can you
think of
a
formula
to find
distance
e.g. KJ ?

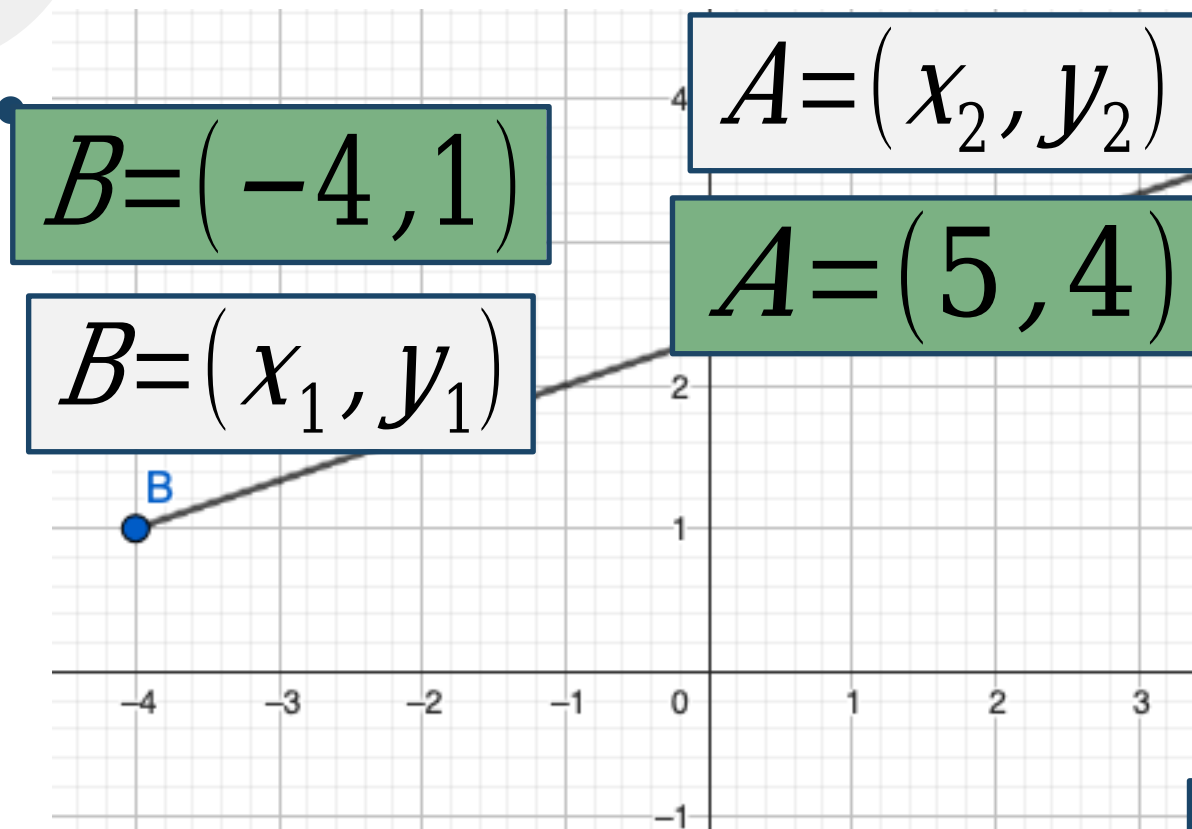
$$KL = \sqrt{i i}$$

This is called the Distance
formula

K can be or but do not mix
them up, e.g.

This is important for
gradient too

What is the distance between and



$$AB = \sqrt{i i}$$

$$AB = \sqrt{i i}$$

$$AB = \sqrt{i i}$$

$$AB = \sqrt{90}$$

$$AB = 9.49 \text{ units}$$

Question 1

- The point A has co-ordinates $(-4, 6)$ and the point B has co-ordinates $(7, -2)$.

[3]

Calculate the length of the line AB .

The point A has co-ordinates $(-4, 6)$ and the point B has co-ordinates $(7, -2)$.

Calculate the length of the line AB .

[3]

The change in x coordinate between points A and B (subtract the coordinates):

$$\Delta x = (-4) - (7) = -11$$

The change in y coordinate between points A and B (subtract the coordinates):

$$\Delta y = (6) - (-2) = 8$$

The length is given as:

$$\text{length} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Use the previous results to get the length of AB .

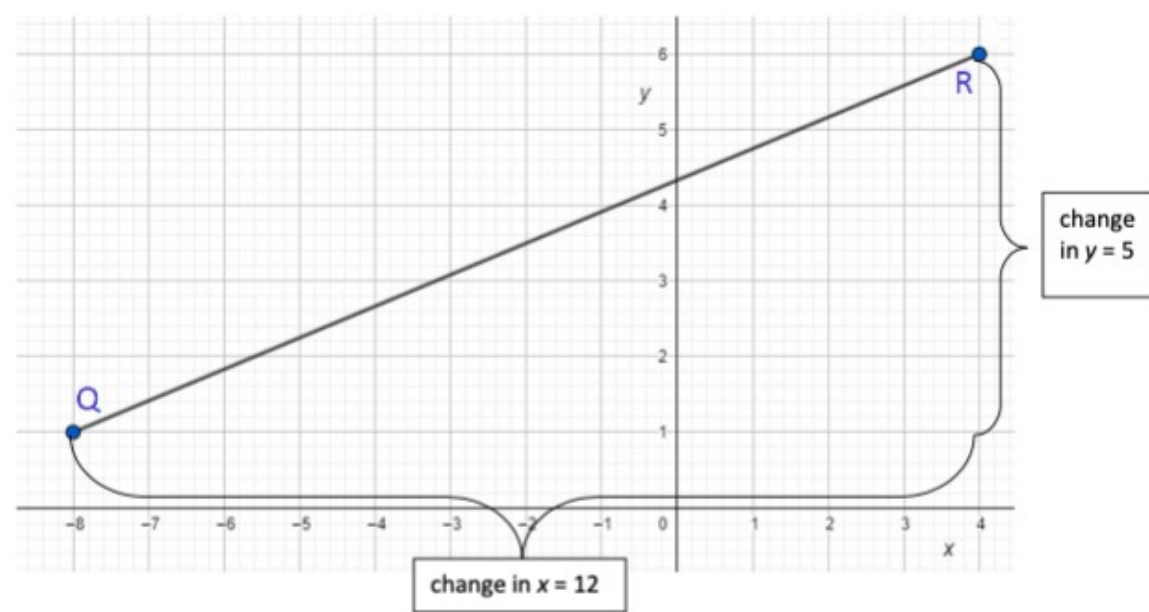
$$\text{length} = \sqrt{(-11)^2 + (8)^2}$$

$$\text{length} = 13.6$$

Question 2

Find the length of the straight line from $Q(-8, 1)$ to $R(4, 6)$.

[3]



The change in x coordinate between points Q and R is found by subtracting the x coordinates:

$$\text{Change in } x = \Delta x = (4) - (-8) = 12$$

The change in y coordinate between points Q and R is found by subtracting the y coordinates:

$$\text{Change in } y = \Delta y = (6) - (1) = 5$$

Using Pythagoras' Theorem, the length is given as:

$$\text{length}^2 = (\Delta x)^2 + (\Delta y)^2$$

$$\text{length} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Use the previous results to get the length of QR.

$$\text{length} = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

$$\text{length} = 13$$

Student activity

- Complete the MathSpace activity –
 - Cartesian Plane – Distance formula –
due Sunday



I will assign MathSpace activities throughout the week and I will check your progress on Sunday

Starter

Given the equation;

- Find the value for y when $x = 0, 1, 2, 3, 4, 5$. Fill out your answers in the table provided.

x	0	1	2	3	4	5
y	1	4	7	10	13	16

For every

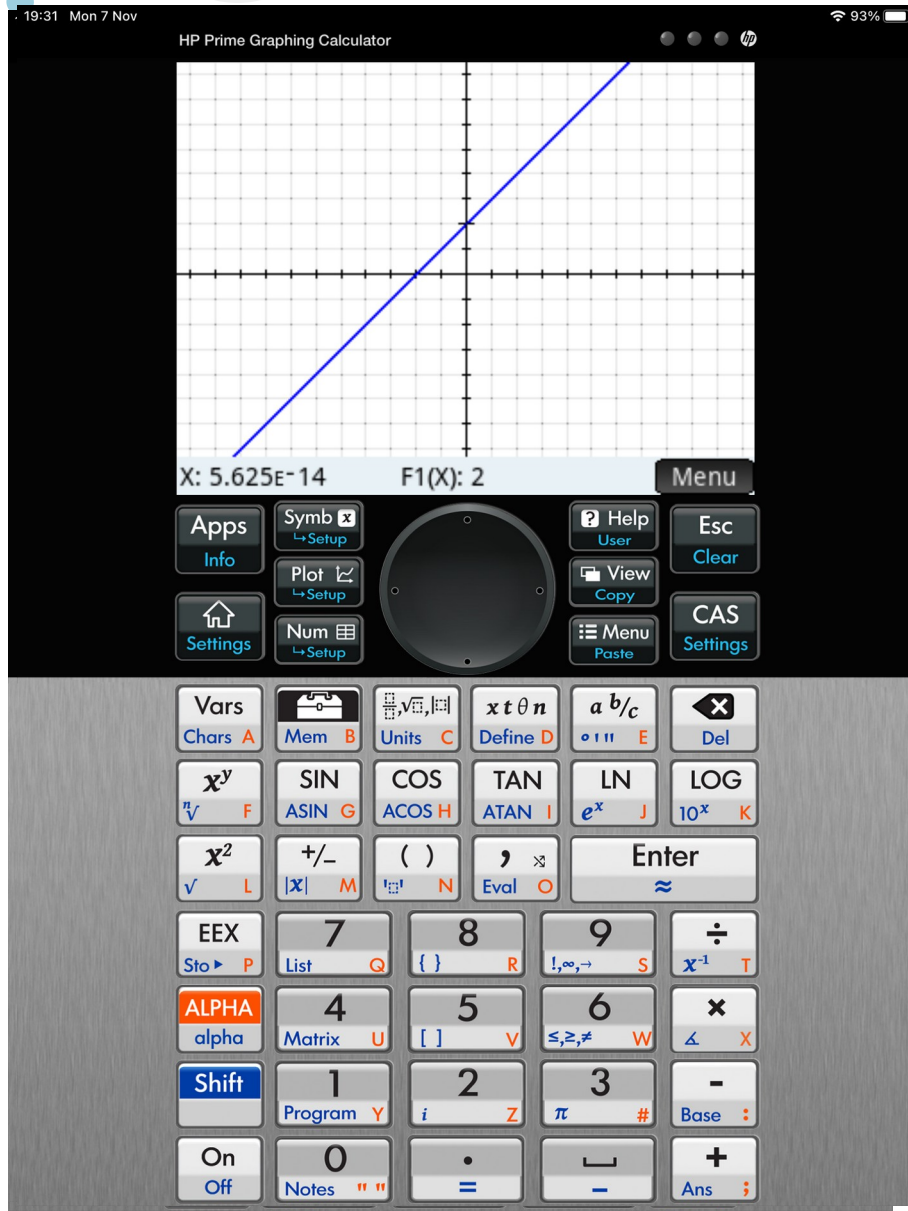
What do you notice about the y values?

Increasing by same amount each time (3)

When $x = 0$, $y = 1$

one increase in x , we got a 3 increase in y

Investigating equation of a straight line



Fill in

Yay!










Plot

**How to draw a
graph on your
GDC**



On your GDC, plot the following straight line graphs and answer the questions on the worksheet

The equation is for a straight line is








Function Symbolic View			18:02
✓		F1(X)=	X-5
✓		F2(X)=	X-4
✓		F3(X)=	X-3
✓		F4(X)=	X-2
✓		F5(X)=	X
✓		F6(X)=	X+2
✓		F7(X)=	X+3
✓		F8(X)=	X+4
✓		F9(X)=	X+5

What do you notice about the graphs if m stays the same, and c changes? What is c ?

On your GDC, plot the following straight line graphs and answer the questions on the

works

The equation is for a straight line is

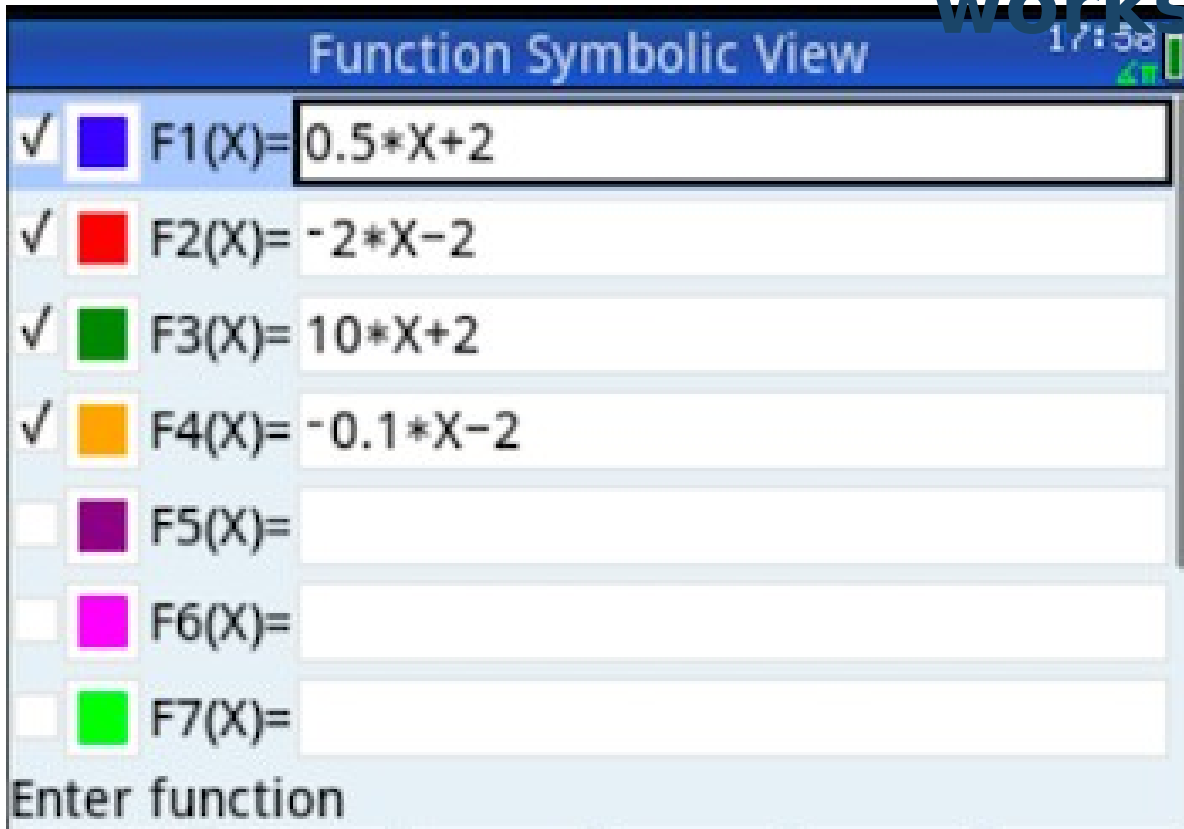
Function Symbolic View		
✓		F1(X)=X
✓		F2(X)= 2*X
✓		F3(X)= 4*X
✓		F4(X)= -2*X
✓		F5(X)= -4*X
✓		F6(X)= -0.5*X
✓		F7(X)= 0.5*X

What do you notice about the graphs if c stays the same, and m changes? What happens when m is positive compared to negative?

On your GDC, plot the following straight line graphs and answer the questions on the

worksheet. The equation is for a straight line is

What do you notice about the first pair of graphs and the second pair of graphs?



Function Symbolic View 17:58

✓	<input checked="" type="checkbox"/>	F1(X)=	0.5*X+2
✓	<input checked="" type="checkbox"/>	F2(X)=	-2*X-2
✓	<input checked="" type="checkbox"/>	F3(X)=	10*X+2
✓	<input checked="" type="checkbox"/>	F4(X)=	-0.1*X-2
	<input type="checkbox"/>	F5(X)=	
	<input type="checkbox"/>	F6(X)=	
	<input type="checkbox"/>	F7(X)=	

Enter function

The equation for a straight line is

What do you notice about the graphs if m stays the same, and c changes? What is c ?

What do you notice about the graphs if c stays the same, and m changes? What happens when m is positive compared to negative?

What do you notice about the first pair of graphs and the second pair of graphs?

Graphs are parallel. C is the y -intercept (where it cuts the y -axis)

Positive - increasing, negative decreasing. As the value of m increases, graph gets more steep

First pair are perpendicular to each other. Second two are perpendicular to each other

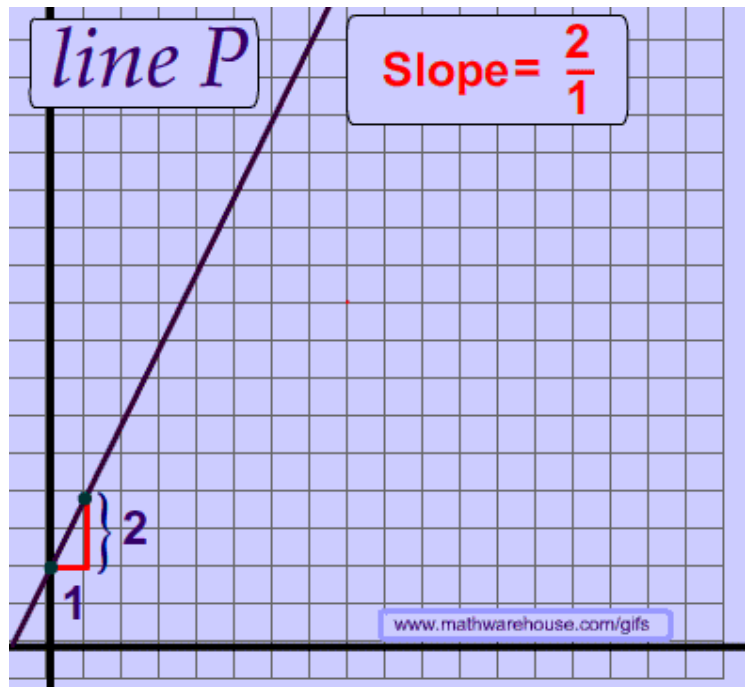
Equation of a straight line

$$y = mx + c$$

Gradient
which is also
described as slope
or rise over run

y-intercept
Where the
graph cuts the
y-axis

Gradient
which is also
described as slope
or rise over run



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Also seen as:

Change in y
(rise)

Change in x
(run)

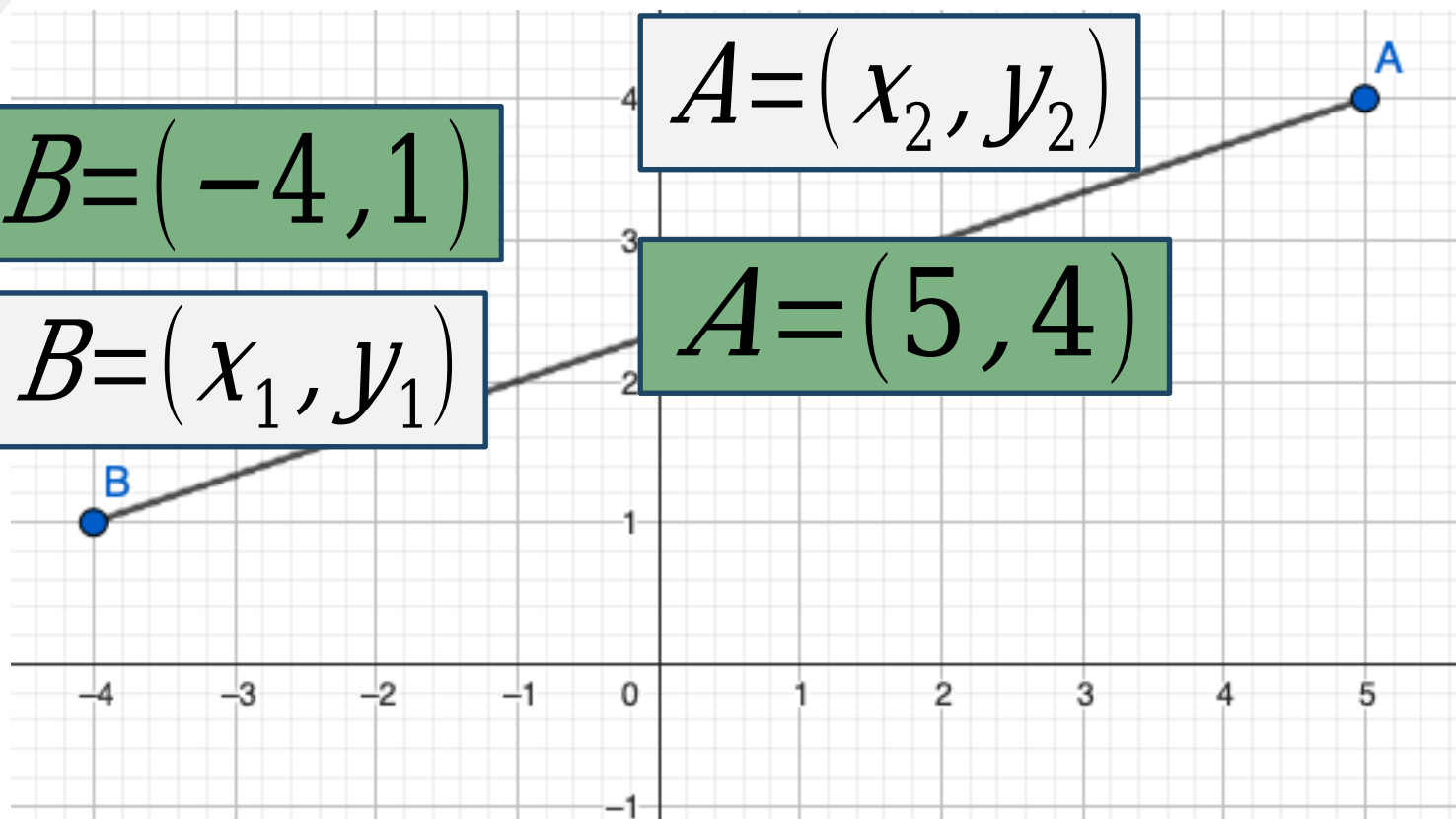
Calculating Gradient

$$B = (-4, 1)$$

$$B = (x_1, y_1)$$

$$A = (x_2, y_2)$$

$$A = (5, 4)$$

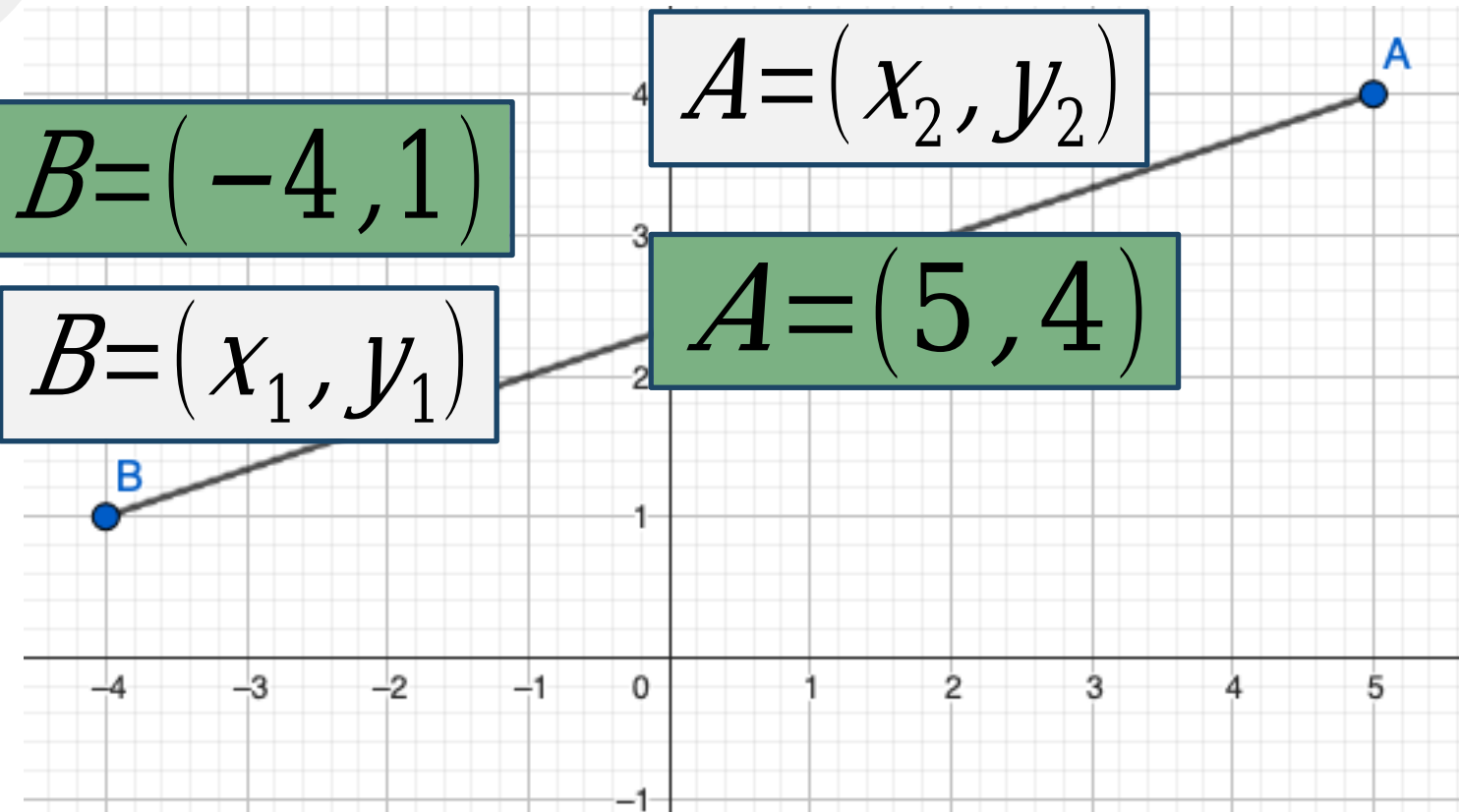


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 1}{5 - (-4)}$$

$$m = \frac{3}{9} = \frac{1}{3}$$

Finding equation of straight line



$$m = \frac{3}{9} = \frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

is the y-intercept,
but I cannot read
it clearly off the
graph □

Finding equation of straight line

$$B = (-4, 1)$$

$$B = (x_1, y_1)$$

$$A = (x_2, y_2)$$

$$A = (5, 4)$$

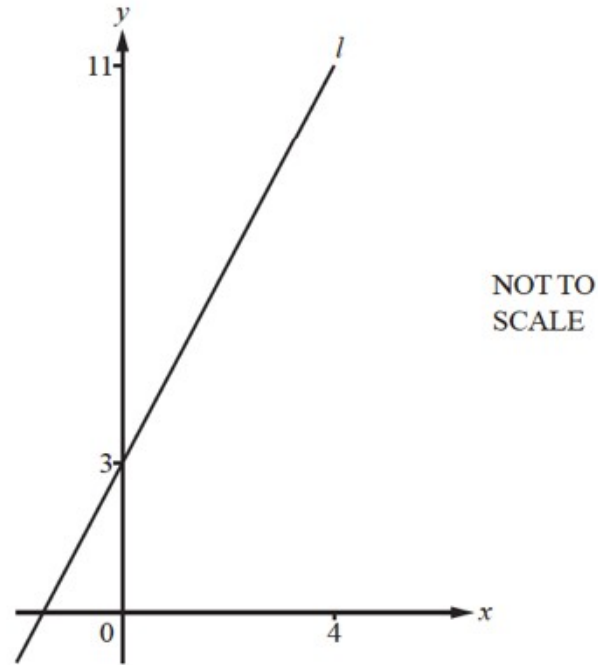
$$y = \frac{1}{3}x + 2\frac{1}{3}$$

$$y = \frac{1}{3}x + c$$

Substitute any value
which is on your line
into the equation
e.g. $(-4, 1)$

$$2\frac{1}{3} = c$$

Question 1



The diagram shows the straight line, l , which passes through the points $(0, 3)$ and $(4, 11)$.

(a) Find the equation of line l in the form $y = mx + c$.

$$y = 2x + 3$$

The general equation of the line is $y = mx + c$. The number m is the gradient

The gradient is found as the change of y-coordinate over the change of x-coordinate

between two points on the line. We have two points (0,3) and (4,11).

$$\text{Gradient } m = \frac{dy}{dx} = \frac{11-3}{4-0} = \frac{8}{4} = 2.$$

To calculate the value of constant c , we simply use one of the points and the equation $y =$

$2x + c$.

Use point (0,3):

$$3 = 2 \times 0 + c$$

$$c = 3$$

The equation of the line: $y = 2x + 3$

Question 2

Find the equation of the line passing through the points with co-ordinates (5, 9) and (−3, 13).

[3]

$$\rightarrow y = -\frac{1}{2}x + \frac{23}{2}$$

Gradient found as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{13 - 9}{-3 - 5}$$

$$= -\frac{4}{8}$$

$$= -\frac{1}{2}$$

Now use straight-line equation

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 13 = -\frac{1}{2}(x + 3)$$

$$\rightarrow y = -\frac{1}{2}x + \frac{23}{2}$$

Question 3

(a) Find the co-ordinates of the midpoint of the line joining $A(-8, 3)$ and $B(-2, -3)$.

[2]

(b) The line $y = 4x + c$ passes through $(2, 6)$.

Find the value of c .

[1]

(c) The lines $5x = 4y + 10$ and $2y = kx - 4$ are parallel.

Find the value of k .

[2]

- (a) Find the co-ordinates of the midpoint of the line joining $A(-8, 3)$ and $B(-2, -3)$.

Midpoint is found as

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)$$

$$= \left(\frac{-8 - 2}{2}, \frac{-3 + 3}{2} \right)$$

$$= (-5, 0)$$

- (b) The line $y = 4x + c$ passes through $(2, 6)$.

Find the value of c .

$$6 = 4(2) + c$$

$$\rightarrow 6 = 8 + c$$

$$\rightarrow c = -2$$

[2]

Where we know that m is the gradient of the line.

$$5x = 4y + 10$$

$$\rightarrow 4y = 5x - 10$$

$$\rightarrow y = \frac{5}{4}x - \frac{10}{4}$$

Hence the gradient of both lines is

$$m = \frac{5}{4}$$

For the other line

$$2y = kx - 4$$

$$\rightarrow y = \frac{k}{2}x - 2$$

Hence

$$\frac{k}{2} = \frac{5}{4}$$

$$\rightarrow k = \frac{5}{2}$$

[1]

Student activity

- Complete the MathSpace activity –
 - Cartesian Plane – Equation of a straight line – *due Sunday*

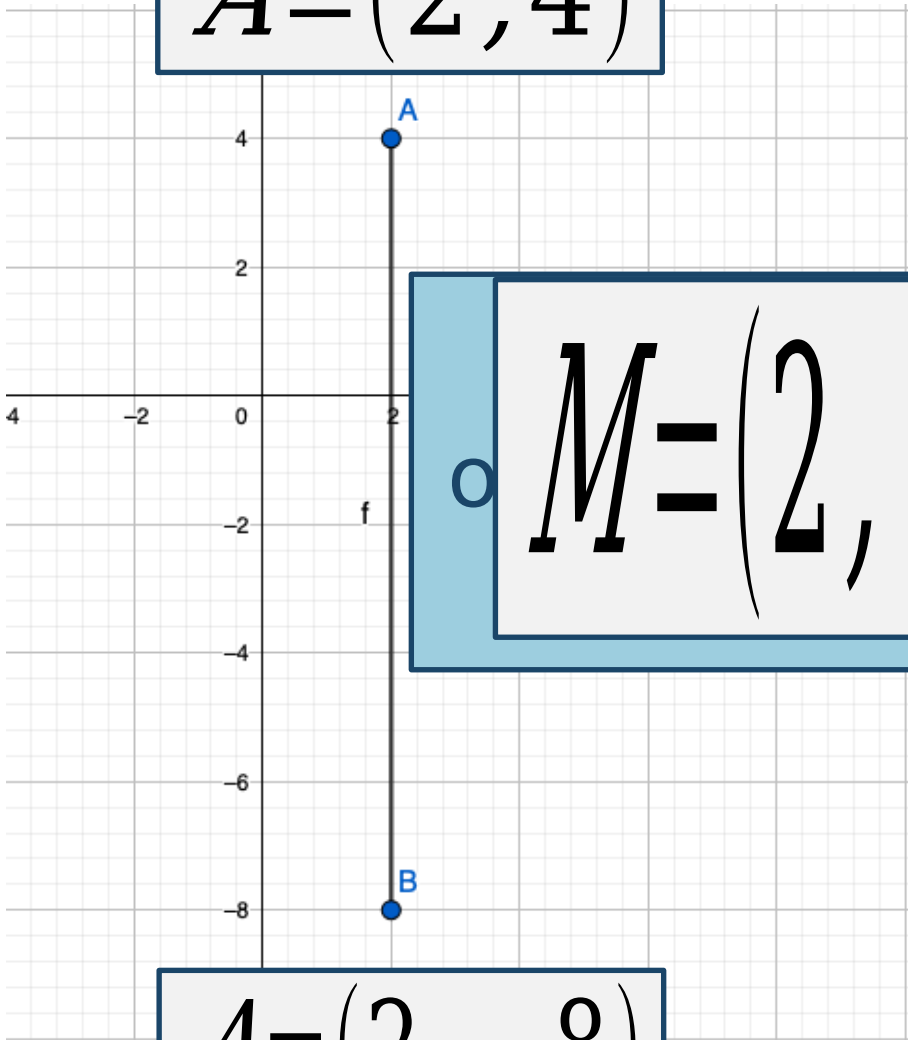


I will assign MathSpace activities throughout the week and I will check your progress on Sunday

Sta

How did you figure out this value?

$$A = (2, 4)$$



$$M = (2, -2)$$

$$A = (2, -8)$$



$$B = (2, 4)$$

$$M = (-2, 4)$$

$$A = (2, 4)$$

$$M = (-4, 0)$$

$$B = (-10, -4)$$

Can you figure out the middle of this line?

How did you figure out this value?

Midpoint formula

,)

Find the mid-point of the two coordinates listed:

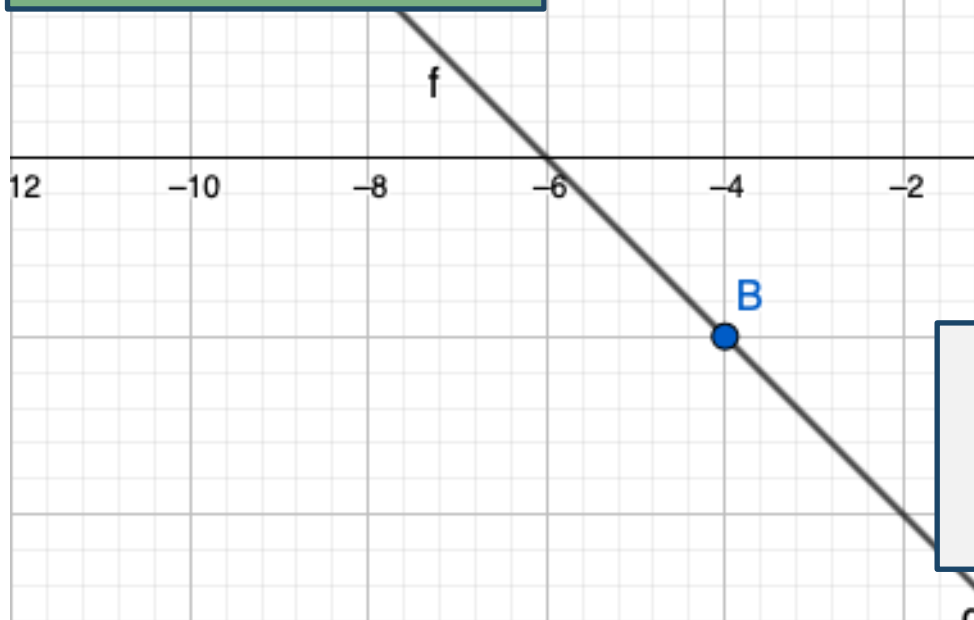
$(-2, 4)$

$(-1, -1)$

$(4.5, 4.5)$

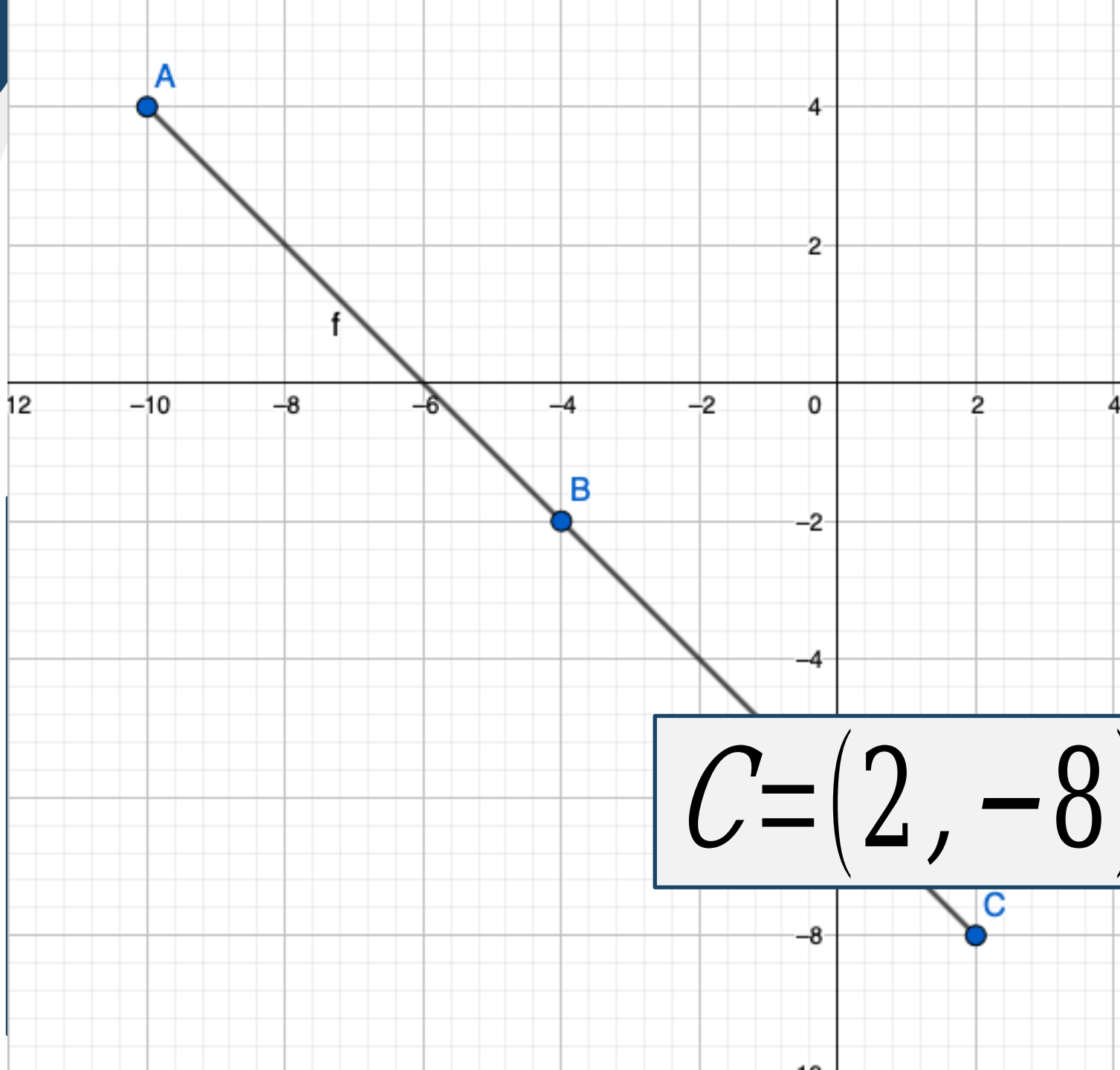
$$A=(x_1,y_1)$$

$$A=(-10,4)$$



$$C=(x_2,y_2)$$

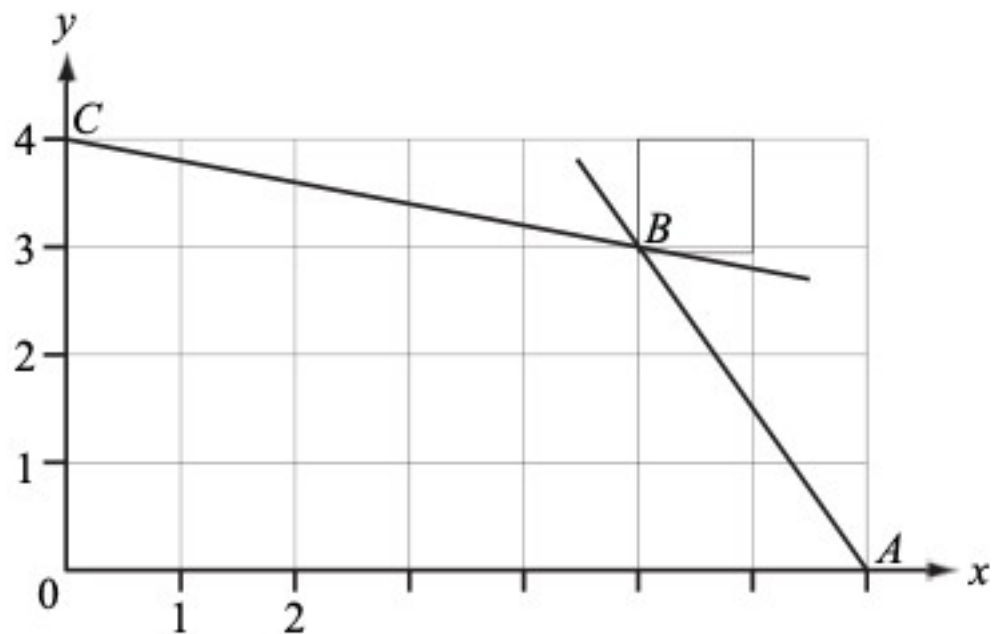
,)



Split into two!

$$C = (2, -8)$$

Example 1



The lines AB and CB intersect at B .

midpoint coordinates = $(6, 1.5)$

(a) Find the co-ordinates of the midpoint of AB .

[1]

(b) Find the equation of the line CB .

$$y = -\frac{1}{5}x + 4$$

[3]

Equations of vertical and horizontal lines

What is

$$y = 4$$

The value for
y is always 4,
no matter
what co-
ordinate you
look at on the

line



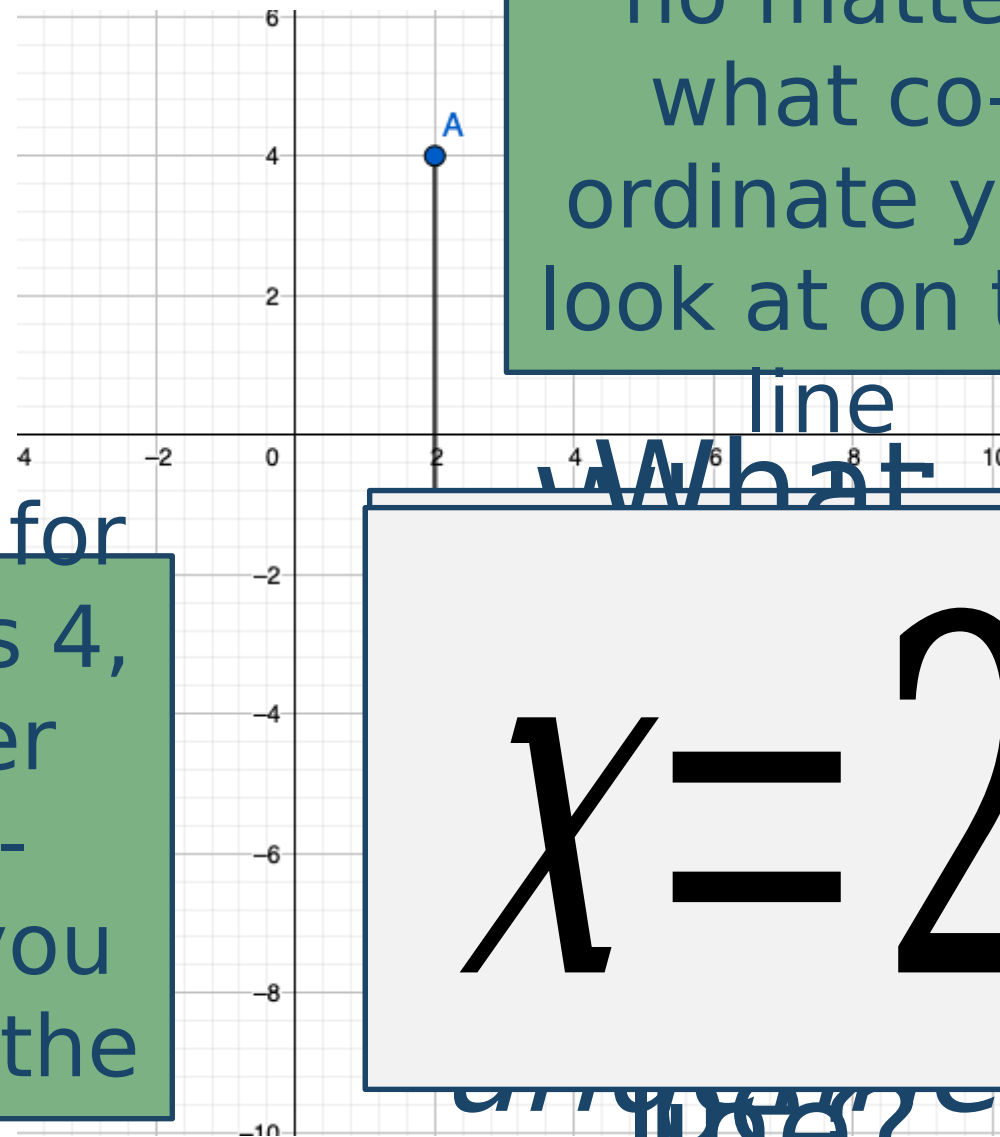
The value for
x is always 2,
no matter
what co-
ordinate you
look at on the

line

What

$$x = 2$$

line?



Student activity

- Complete the 5 General practice PPQs
- Complete the MathSpace activities if you have not done so already



Well done!



Everyone.... **EVERYONE...** Did both MathSpace activities! Almost everyone pressed 'Hand in' on Classroom to let me know.

2 HOUSE POINTS TO EVERYONE!

This week I will be looking for great participation and consistent English use



$$f : y = x$$



$$g : y = 2x + 2$$



$$h : y = -x$$



$$p : y = 4x - 4$$



$$q : y = 0.5x - 3$$



$$r : y = 1$$



$$\text{eq1} : x = -4$$

Starter

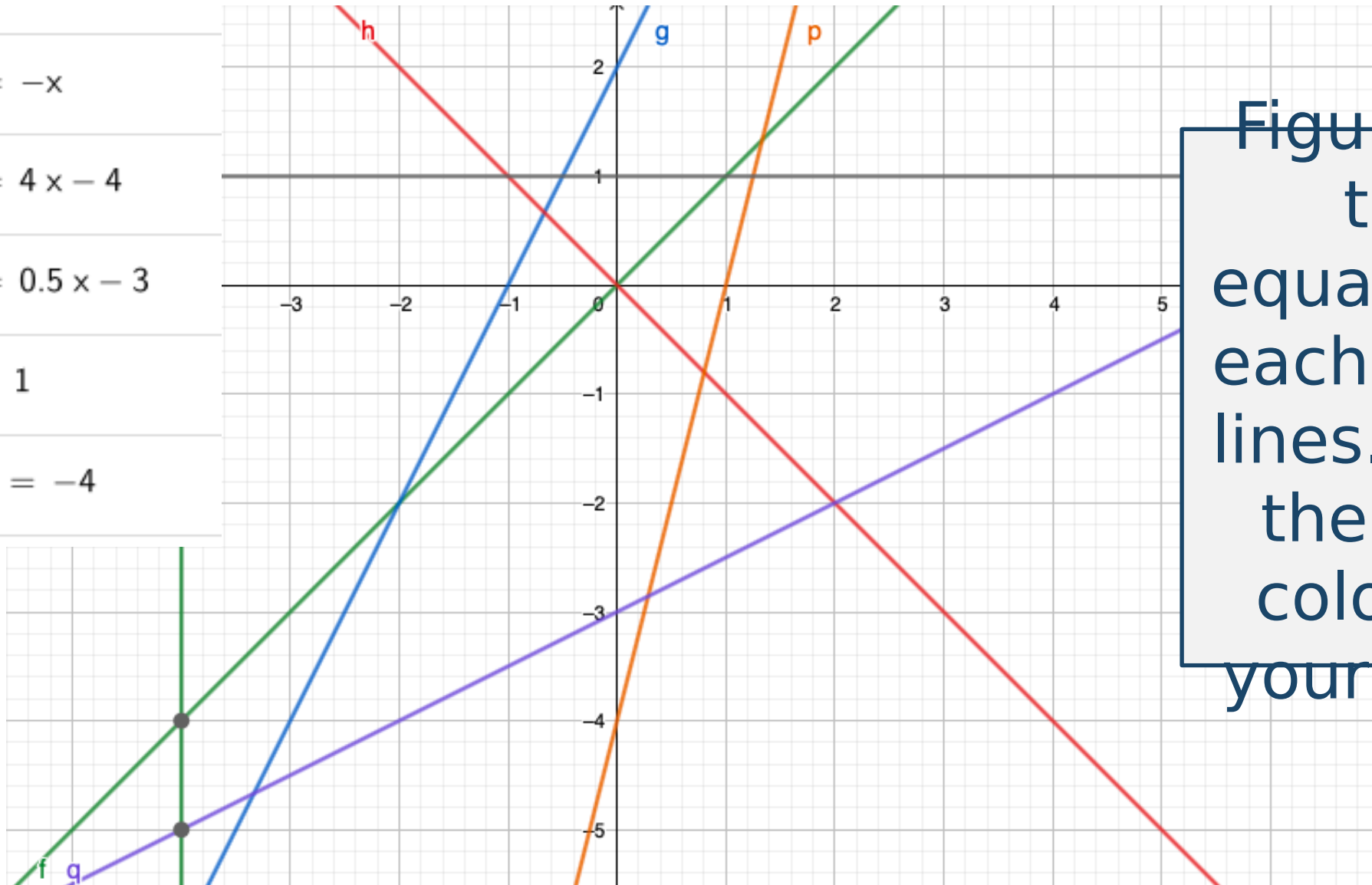
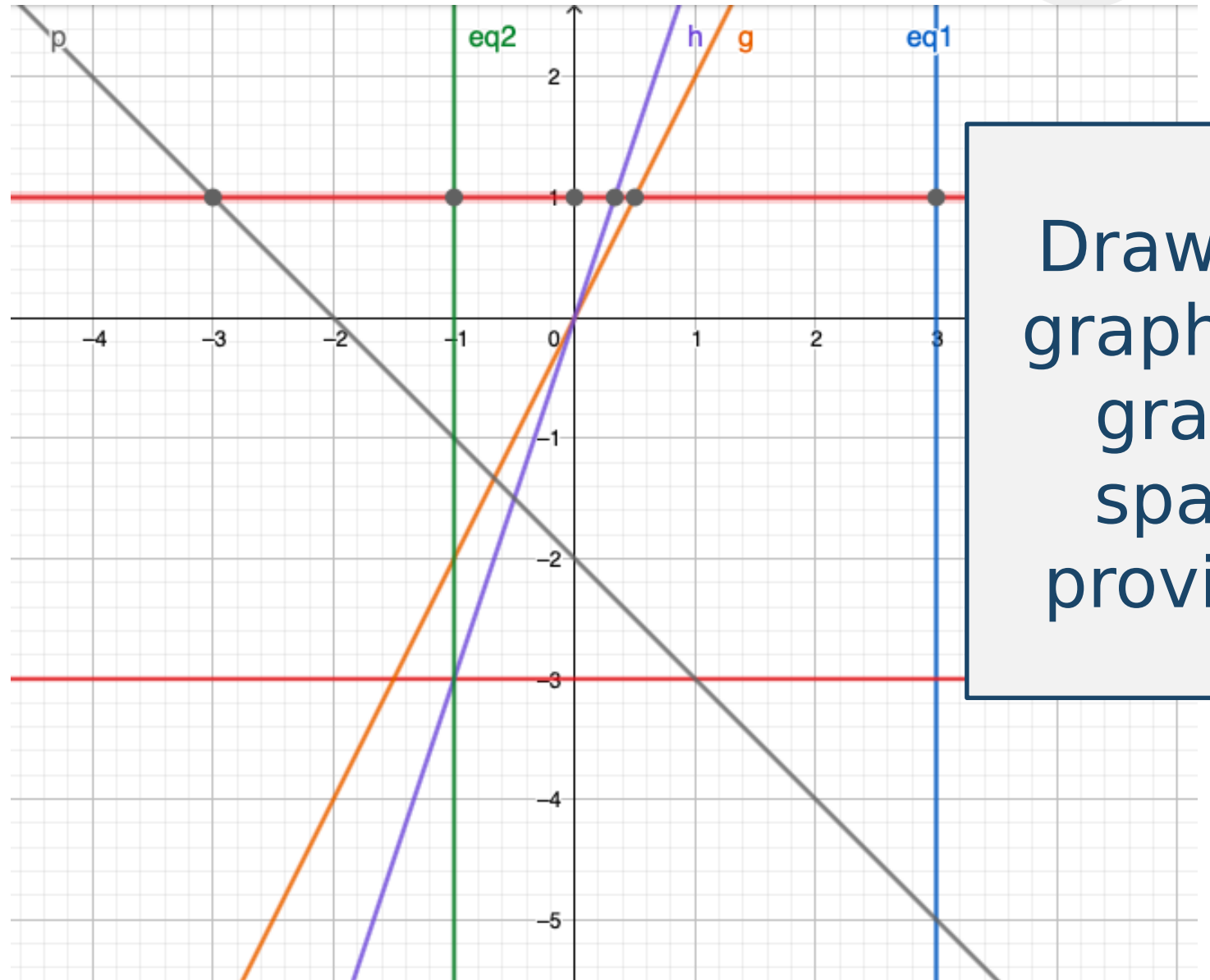


Figure out
the
equation of
each of the
lines. Label
them by
colour in
your notes

Starter

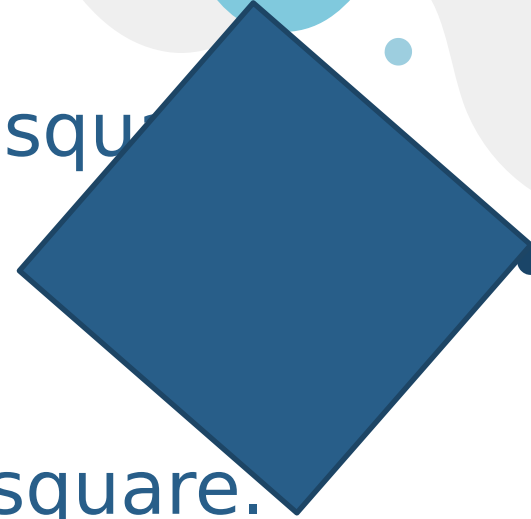
●	eq1 : $x = 3$
●	f : $y = -3$
●	g : $y = 2x$
●	h : $y = 3x$
●	p : $y = -x - 2$
●	eq2 : $x = -1$
+	$y = 1$



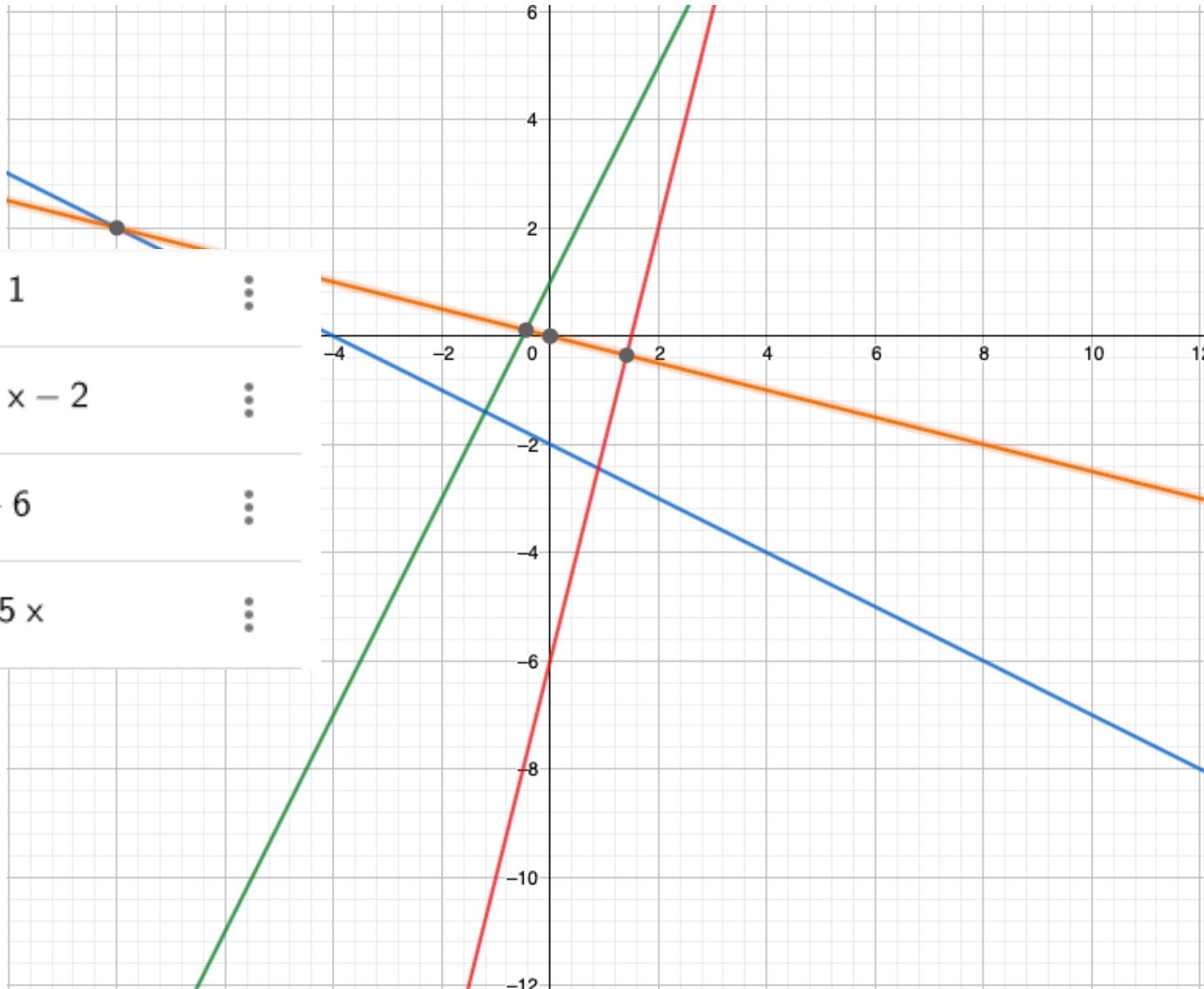
Draw the
graphs on
graph
space
provided

On graph paper provided

- Plot 4 co-ordinates that if joined would form a square.
Make sure your square is at an angle.
- Join the lines to form a square.
- Label each corner with a letter (A, B, C, D)
- Find the equation of each of the sides of your square.
What do you notice about the gradients?
- Find the distance of each of the sides (hopefully you actually drew a square!)
- Draw in the two diagonals of the square. Find the midpoint of each of those lines. What do you notice?
- Find the equations of your diagonals. What do you notice about their gradients?



Starter



Find the equations of the lines and label them by colour

Can you spot
a trick to
check
yourself?

Flip your fraction,
change the sign!

For e.g. if line A has
a gradient of what
will the gradient be
of the
perpendicular line

Complete this table in your notes

Plot		Distance	Midpoint	Equation of line	Equation of line perpendicular to, and through midpoint
<div>●</div> <div>●</div> <div>●</div>	A = (-6, 4)	AF			
	B = (4, 4)	CB			
	C = (-10, -4)				
●	D = (2, -8)	ED			
●	E = (12, 6)	AD			
●	F = (-4, -4)				

Use the back the of the note for your calculations

Finished early? Try drawing the straight lines on your GDC and seeing if they are actually perpendicular

This is called the equation of the perpendicular bisector

Student activity

- Complete the easy and medium questions at

https://www.savemyexams.co.uk/igcse/maths_extended/cie/23/topic-questions/2-algebra-and-graphs/2-12-coordinate-geometry/-/paper-2-and-paper-4/easy/



If you have your GDC, try figuring out the hard one!

Check 5 questions from last Friday

- Open your laptops, go to classroom and open the presentation called Y10 Cartesian Planes. Go through the answers for Questions 1 – 5. You may discuss quietly with the person next to you.
- In 15 minutes, we will go through some of the questions together.
- Keep in mind – these are past paper questions and they might be difficult at the moment. After some practice it gets easier.

Answers from PPQs

The change in x coordinate between points Q and R is found by subtracting the x coordinates:

$$\text{Change in } x = \Delta x = (4) - (-8) = 12$$

The change in y coordinate between points Q and R is found by subtracting the y coordinates:

$$\text{Change in } y = \Delta y = (6) - (1) = 5$$

Using Pythagoras' Theorem, the length is given as:

$$\text{length}^2 = (\Delta x)^2 + (\Delta y)^2$$

$$\text{length} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Use the previous results to get the length of QR.

$$\text{length} = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

$$\text{length} = 13$$

Question 1

Question 2

- (a) Write down the equation of the line through B which is parallel to $y = 2x + 3$. [2]

The y -coordinate of A is the y -intercept of the straight line $y = 2x + 3$, which is 3.

B is 7 below A

$$3 - 7$$

$$= -4$$

$$\rightarrow B = (0, -4)$$

The line is parallel (has the same gradient) and cuts through B , giving us

$$y = 2x - 4$$

- (b) Find the co-ordinates of the point C where this line crosses the x axis. [1]

Need to solve

$$0 = 2x - 4$$

$$\rightarrow 2x = 4$$

$$\rightarrow x = 2$$

$$\rightarrow C = (2, 0)$$

(a) Find the gradient of the line as a fraction in its simplest form.

[1]

Gradient is found as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence

$$m = \frac{0 - 8}{10 - 0}$$

$$= -\frac{4}{5}$$

Question 3

(b) **Write down** the equation of the line parallel to l which passes through the origin.

[1]

Parallel so it has the same gradient.

Passes through the origin so y intercept is zero

$$\rightarrow y = -\frac{4}{5}x$$

(c) Find the equation of the line parallel to l which passes through the point $(3, 1)$.

[2]

Use the straight-line equation

$$y - y_1 = m(x - x_1)$$

$$\rightarrow y - 1 = -\frac{4}{5}(x - 3)$$

$$\rightarrow 5y - 5 = -4x + 12$$

$$\rightarrow 4x + 5y - 17 = 0 \text{ or } y = -\frac{4}{5}x + 3.4$$

Question 4

The equation of a straight line can be written in the form $3x + 2y - 8 = 0$.

- (a) Rearrange this equation to make y the subject.

[2]

$$3x + 2y - 8 = 0$$

$$2y = 8 - 3x$$

$$y = \frac{8 - 3x}{2}$$

- (b) Write down the gradient of the line.

[1]

The equation of a line takes the form: $y = mx + n$

Where m is the gradient and n is the y -intercept.

In our case, the equation of the line from a) is:

$$y = 4 - \frac{3x}{2}$$

The gradient is: $m = -\frac{3}{2}$

- (c) Write down the co-ordinates of the point where the line crosses the y axis.

[1]

The point where the line crosses the y axis has the x coordinate $x = 0$.

We substitute x in the equation to work out y .

$$y = \frac{8-3x}{2}$$

$$8 - 3 \times 0 = 2y$$

$$2y = 8$$

$$y = 4$$

Coordinate: **(0, 4)**

Question 5

The straight line graph of $y = 3x - 6$ cuts the x -axis at A and the y -axis at B .

- (a) Find the coordinates of A and the coordinates of B .

[2]

Point A is the intersection of the graph with the x -axis, therefore, $y = 0$.

$$0 = 3x - 6$$

$$x = 2$$

$$A = (2, 0)$$

Point B is the intersection of the graph with the y -axis, therefore, $x = 0$.

$$y = 3 \times 0 - 6$$

$$y = -6$$

$$B = (0, -6)$$

(b) Calculate the length of AB .

$A (2, 0), B (0, -6)$

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

$$AB = \sqrt{(0 - 2)^2 + ((-6) - 0)^2}$$

$$AB = \sqrt{4 + 36}$$

$$AB = \sqrt{40} = 6.32$$

[2]

- (c) M is the mid-point of AB .
Find the coordinates of M .

[1]

M is the midpoint of AB .

$$x_M = \frac{x_A + x_B}{2} = \frac{2 + 0}{2} = 1$$

$$y_M = \frac{y_A + y_B}{2} = \frac{0 + (-6)}{2} = -3$$

$M(1, -3)$